6. What Makes Resistors Hot?

As we all know, power dissipation makes resistors hot. How hot they get is a subject to be discussed later. Why they get hot is what this chapter's all about.

Let's start with a problem. Consider Fig. 6-1. In it we see a condensed version of part of the Fig. 1-1 block diagram. It shows only the high-voltage power supply, the energy-storage capacitor bank, a load (in the form of a diode-gun microwave-amplifier tube), a pulse modulator (in the form of a single-pole, single-throw switch), and two resistors, one between the power supply and capacitor bank and the other between the capacitor bank and load.

The current through the load is pulse-modulated by opening and closing the switch. When the switch is closed for interval T_{on}, the current, I_2, which is limited by the diode load, is I_{peak}. This is also the current flowing through the resistor *R2*. During the interpulse interval, shown as T_{off}, the switch is open. The current during this interval is zero. The pulses recur at a constant rate called the pulse-repetition frequency (PRF). The interval between the leading edges of successive pulses is called the pulse-repetition interval (PRI). The duty factor of this waveform is the ratio of the "on" interval to the PRI. Because our capacitor C_{huge} is arbitrarily large, and because our waveform has presumably been going since the beginning of time, it is safe to assume that all of the current delivered to the load during the pulse came from the storage capacitor. It is also safe to assume that the current from the power supply is continuous, non-variant, and equal to the time-averaged value of the pulse current, which is the product of I_{peak} times the duty factor. These conditions, and the fact that there can be no average current

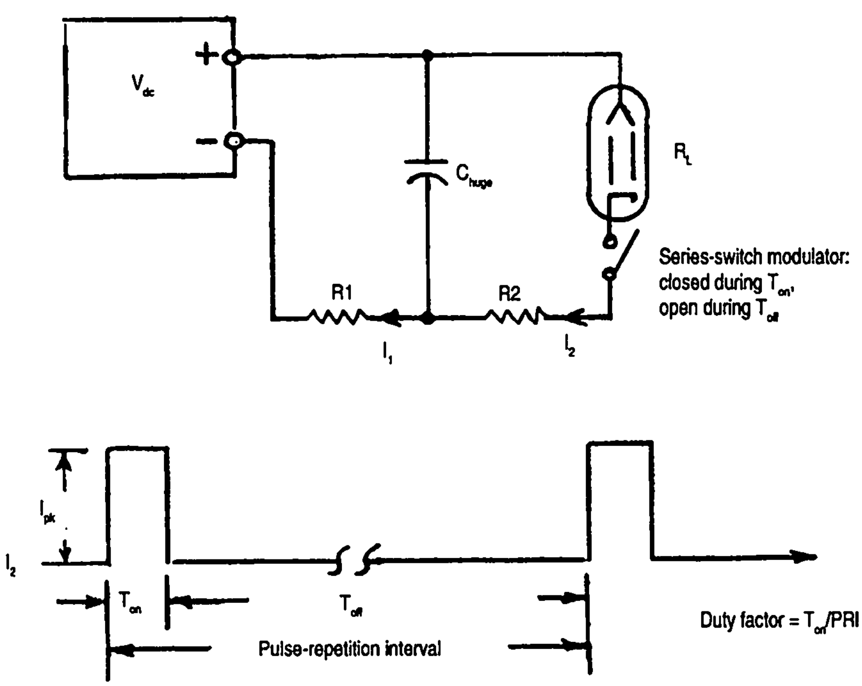

Figure 6-1. Which resistor gets hotter?

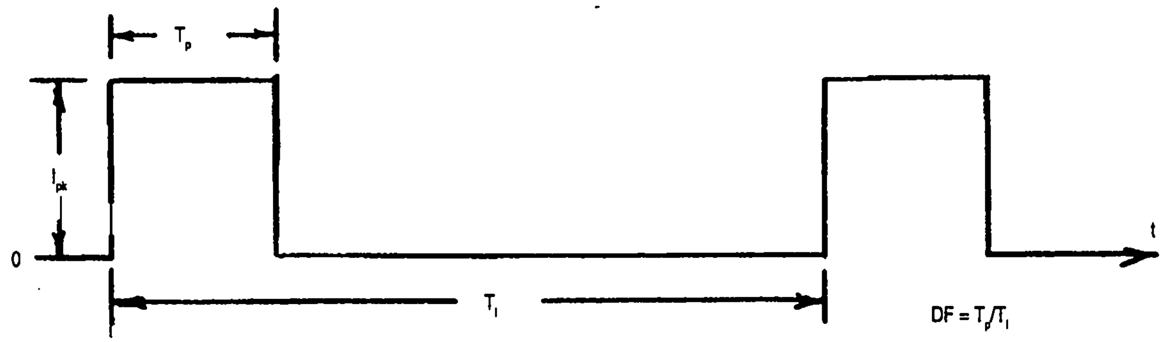

Figure 6-2. Recurrent rectangular waveform.

through a capacitor in any case, mean that the average current is the same in *R1* and *R2*. Now, here's the problem: if they both have the same resistance, which one gets hotter? (You might be amazed at how many engineers do not know the correct answer, even if you do.)

The fact that the average current through both resistors is the same does not mean that the power dissipation is the same, because average current is not what causes power-dissipation in resistors. (If it did, pure alternating current would cause no dissipation, because its average value is zero.) Average current is what causes power dissipation in unidirectional, non-linear devices like semiconductor diodes and ionized plasma. These devices tend to have a voltage drop that, to a first approximation, is independent of current. What causes power dissipation in resistors is root-mean-square (RMS) current. Unlike any other circuit element, the voltage across a resistor is proportional to and in phase with the current through it. Power dissipation, then, will be proportional to the square of the current. Therefore, *R2* will always get hotter than *R1*—unless the duty factor reaches unity, at which point the current flowing through both resistors will be unidirectional and non-variant, which is the only wave shape that has identical RMS and average values. The term dc, by the way, is not adequate to define a non-variant unidirectional current, because the wave shape in Fig. 6-1 is dc for all duty factors. The current still only goes in one direction, never reversing. (Do not be fooled by the Fourier series, which correctly characterizes the rectangular-pulse periodic waveform as an average term and an infinite series of sinusoidal ac terms that are harmonically related to the PRF. They all exist simultaneously, and their sum is unidirectional.)

Figure 6-2 shows a recurrent rectangular pulse waveform. The time-averaged and RMS values for the current can be found by using these simple equations:

$$I_{avg} = \frac{\int_0^{T_p} I_{pk}\,dt}{T_i} = \frac{I_{pk}T_p}{T_i} = I_{pk}DF$$

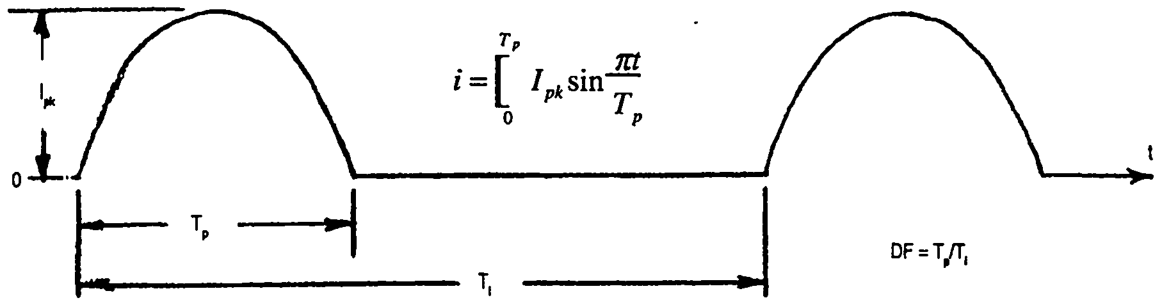

Figure 6-3. Recurrent rectified sine wave.

$$I_{RMS} = \sqrt{\frac{\int_0^{T_p} I_{pk}^2 dt}{T_i}} = \sqrt{I_{pk}^2 \frac{T_p}{T_i}} = I_{pk}\sqrt{\frac{T_p}{T_i}} = I_{pk}\sqrt{DF}$$

$$\frac{I_{RMS}}{I_{avg}} = \frac{\sqrt{DF}}{DF} = \frac{1}{\sqrt{DF}}$$

$$\frac{I_{RMS}^2}{I_{avg}^2} = \frac{1}{DF}.$$

The important thing to note is that the ratio of RMS and average current varies inversely as the square root of the duty factor. Even more importantly, the square of this ratio, which is directly proportional to heating, varies as the inverse of the duty factor. This means in our example of Fig. 6-1 that if the duty factor had been 0.001 (or 0.1%)—not an uncommon duty factor for simple pulse radar systems— resistor *R*2 would have to dissipate 1000 times as much power as *R*1.

Figures 6-3, 6-4, and 6-5 show the derivations for the corresponding values for the recurrent rectified sine wave, the recurrent triangular waveform, and the recurrent exponential decay waveforms, respectively.

Refer to Fig. 6-3 for the following derivation of RMS and average current values:

$$I_{avg} = \frac{T_p}{\pi T_i}\int_0^{T_p} I_{pk} \sin\frac{\pi t}{T_p} dt = \frac{T_p I_{pk}}{T_i \pi}\left[-\cos\frac{\pi t}{T_p}\right]_0^{T_p}$$

$$= \frac{T_p I_{pk}}{T_i \pi}[-\cos\pi - (-\cos 0)] = \frac{T_p I_{pk}}{T_i \pi}[-(-1)-(-1)] = \frac{2T_p I_{pk}}{\pi T_i}$$

$$= \frac{2}{\pi} I_{pk} DF$$

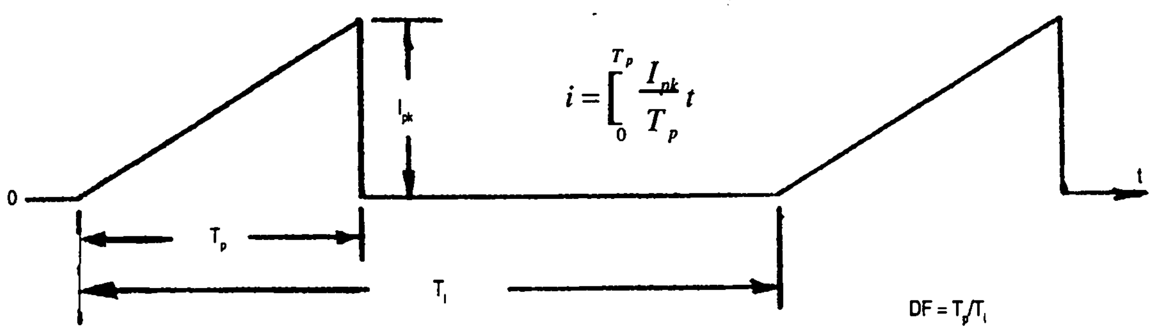

Figure 6-4. Recurrent triangular waveform.

$$I_{RMS} = \sqrt{I_{pk}^2 \frac{T_p}{\pi T_i} \int_0^{T_p} \sin^2 \frac{\pi t}{T_p} dt} = I_{pk} \sqrt{\frac{T_p}{\pi T_i} \frac{1}{2} \left[_0^{T_p} \frac{\pi t}{T_p} - \sin \frac{\pi t}{T_p} \cos \frac{\pi t}{T_p}\right.}$$

$$= I_{pk} \sqrt{\frac{T_p}{T_i} \frac{\pi}{2\pi}} = \frac{I_{pk}}{\sqrt{2}} \sqrt{\frac{T_p}{T_i}}$$

$$= \frac{I_{pk}}{\sqrt{2}} \sqrt{DF}$$

$$\frac{I_{RMS}}{I_{avg}} = \frac{\pi}{2\sqrt{2}\sqrt{DF}} = \frac{1.11}{\sqrt{DF}}$$

$$\frac{I_{RMS}^2}{I_{avg}^2} = \frac{1.23}{DF}.$$

Note that at unity duty factor the average value is the familiar $2/\pi$ times the peak value, and the RMS value is the even more familiar peak value divided by the square root of 2. Even at unity duty factor, however, the ratio of RMS to average current is greater than unity because the rectified sine wave is not a constant value during its "on" interval.

Figure 6-4 shows the recurrent triangular wave, which is even farther away from a constant value during the "on" time than the rectified sine wave. The triangular wave manifests an even greater ratio of RMS-to-average current values. The derivation of these values is as follows:

$$I_{avg} = \frac{1}{T_i} \int_0^{T_p} \frac{I_{pk}}{T_p} t \, dt = \frac{I_{pk}}{T_i} \left[_0^{T_p} \frac{I_{pk} t^2}{2t_p}\right. = I_{pk} \frac{T_p^2}{2T_i T_p} = \frac{I_{pk} T_p}{2T_i}$$

$$= \frac{I_{pk}}{2} DF$$

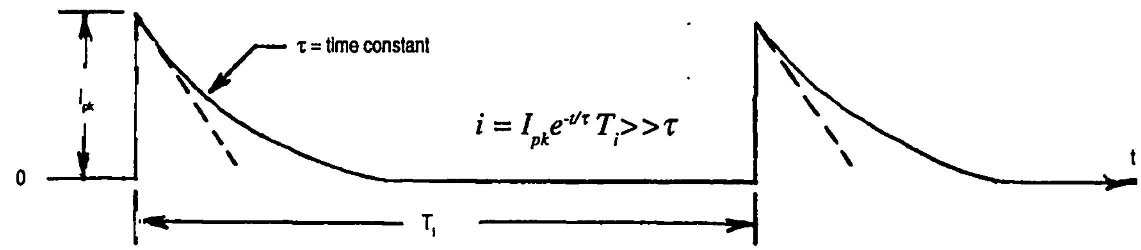

Figure 6-5. Recurrent exponential waveform.

$$I_{RMS} = \sqrt{\frac{1}{T_i} I_{pk}^2 \int_0^{T_p} \frac{t^2}{T_p^2} dt} = \sqrt{\frac{1}{T_i} I_{pk}^2 \left[\frac{t^3}{3T_p^2} \right]_0^{T_p}} = I_{pk}\sqrt{\frac{T_p^3}{3T_iT_p^2}} = \frac{I_{pk}}{\sqrt{3}}\sqrt{\frac{T_p}{T_i}}$$

$$= \frac{I_{pk}}{\sqrt{3}}\sqrt{DF}$$

$$\frac{I_{RMS}}{I_{avg}} = \frac{2}{\sqrt{3}\sqrt{DF}} = \frac{1.555}{\sqrt{DF}}$$

$$\frac{I_{RMS}^2}{I_{avg}^2} = \frac{1.33}{DF}.$$

The waveform of Fig. 6-5, the recurrent exponentially decaying waveform, is also quite common in transmitter-type circuits. It has no "on" time as such. The decay time-constant plays a similar role. The RMS-to-average current ratios shown are true as noted only for repetition intervals that are very large with respect to the decay time-constant. The derivation of their values is as follows:

$$I_{avg} = \frac{1}{T_i}\int_0^{T_i} I_{pk}e^{-t/\tau}dt = \frac{\tau}{T_i}\left[I_{pk}e^{-t/\tau} \right]_0^{T_i} = I_{pk}\frac{\tau}{T_i}$$

$$I_{RMS} = \sqrt{\frac{1}{T_i}\int_0^{T_i} I_{pk}^2 e^{-2t/\tau}dt} = \sqrt{\frac{\tau}{2T_i} I_{pk}^2} = I_{pk}\sqrt{\frac{\tau}{2T_i}}$$

$$\frac{I_{RMS}}{I_{avg}} = \sqrt{\frac{T_i}{2\tau}}$$

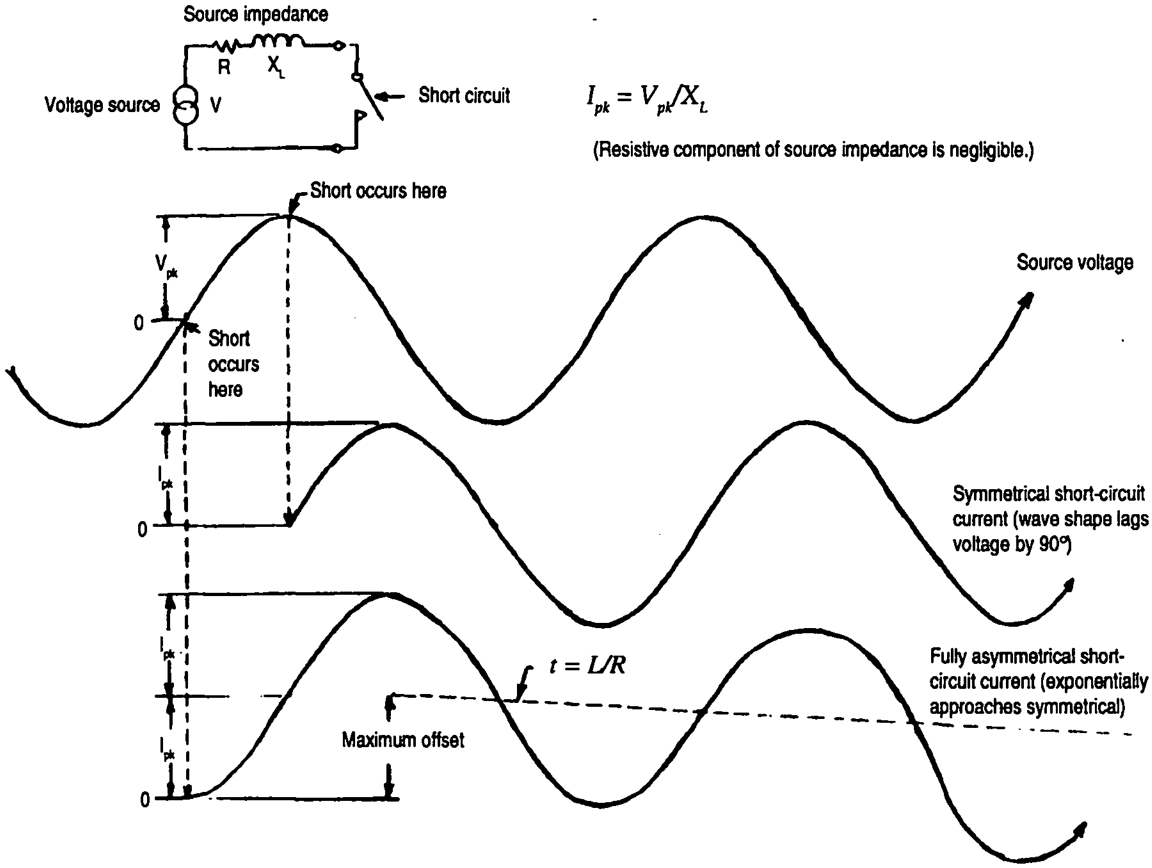

Figure 6-6. RMS values for short-circuit currents in ac circuits.

$$\frac{I_{RMS}^2}{I_{avg}^2} = \frac{T_i}{2\tau}.$$

The waveforms of Fig. 6-6 are especially interesting for a couple of reasons. Unlike the others, they are ultimately ac waveforms. They result from short-circuiting an ac source, which is exactly what unintentional flash arcs in the high-power microwave tube or intentional breakdowns of an electronic crowbar switch will do to the ac primary power of the high-power transmitter shown in Fig. 1-1.

Where the RMS value of the ac component can be given as

$$\frac{I_{pk}}{\sqrt{2}}$$

and where the RMS value of maximum offset is I_{pk}, then the initial RMS value of asymmetrical current is

$$I_{RMS} = I_{pk}\sqrt{I^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = I_{pk}\sqrt{\frac{3}{2}}\ .$$

The magnitude of the short-circuit current is limited by the reactance of the source inductance, which is derived predominantly from the leakage reactance of the step-up and/or step-down transformers that precede the high-voltage rectifier assembly. Even though there will always be series resistance as well, when it is combined as the square root of the sum of the squares with the inductive reactance, it invariably has a negligible contribution to the total source impedance. (The resistive component *will* have an effect on the nature of the current waveform, however, as we shall see.)

Have you ever watched fuses blow? If you have, you have probably wondered why sometimes the event is accompanied by a flash of light that results in the fuse's glass enclosure being completely blackened on the inside and other times, under what appear to be the same conditions, the event causes only a tiny break in the fuse link. The difference may not be due to any physical difference between the fuses. It may be due to the nature of the current flowing through it.

As mentioned before, the short-circuit current will be limited by the source inductive reactance, which means that it will also lag the source voltage by 90° yet have the same wave shape as the voltage did 90° earlier. Suppose the arc that short circuits our system occurs when the supply voltage is at an instantaneous maximum. The short-circuit current will commence with the same wave shape that the voltage had 90° earlier when it was passing through the zero-voltage point. The current wave shape will be sinusoidal and symmetrical until the fast-interrupt switch clears the line. The peak value of current will be the peak value of voltage divided by the source inductive reactance and will generally be 10 to 15 times greater than the normal full-load current. But what if the short circuit does not occur at a voltage maximum? Suppose, instead, that it occurs at a voltage minimum, when it is passing through zero. The short-circuit current will once again commence with the wave shape that the voltage had 90° earlier, which, in the case illustrated, was a negative peak. If the current has this shape starting from zero, it will continue to grow as a cosine wave until its instantaneous maximum is twice the peak value. This short-circuit current is fully asymmetrical and has a unidirectional maximum offset equal to the peak value of current. This offset component will exponentially decay with a time-constant of L/R, where L and R are the source inductance and resistance, respectively. We know what the RMS value of the symmetrical current is: the peak value divided by the square root of 2.

At the beginning of the fault current, the fully asymmetrical waveform is a composite of a constant-value dc term, which is equal to the peak value of symmetrical current and the symmetrical current itself. This condition is especially true if the L/R time-constant is long. The RMS value of this or any other composite waveform comprising components at different frequencies can be evaluated by taking the square root of the sum of the squares of the RMS values of the individual components. For this waveform the value is

$$I_{pk}\sqrt{\frac{3}{2}},$$

which is $\sqrt{3}$ times as large as the RMS value of the symmetrical waveform. This means that the initial power dissipation in a fuse, or the initial magnetic force in a circuit breaker, can differ by a factor of 3, depending on when in the voltage cycle the short circuit occurs. That's one reason why the same type of fuse can sometimes put on a show when it blows and at other times give up the ghost quietly.