Measurement of quasistatic Maxwell's displacement current

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We have used a superconducting quantum interference device to measure all components of the small magnetic field inside a coaxial capacitor that is being charged at a low frequency (695 Hz). The expected result is that B_z and B_ρ should be 0 and that B_ϕ should increase linearly from the end of the capacitor. This prediction depends upon Maxwell's displacement current $\mathbf{J}_D$ being strictly equal to $(1/4\pi)\partial\mathbf{D}/\partial t$ in the annular region between the electrodes. Anomalous fields would arise if J_D twisted a little in going from one electrode to the other. We find no evidence for such a twist. B_ϕ is as predicted; B_ρ and $B_z=0$ everywhere to within a few percent of B_ϕ. Our results show that the displacement current flows within $\Delta\phi=22°$ of the radial direction. We have also searched for a B field that is in phase with the charge (rather than the current) and for fields which vary as the second harmonic of the charging frequency. The former search is interpreted to show that in a traveling TEM wave B is perpendicular to E to within 0.8×10^{-8} rad. The latter search is loosely connected to nonlinear electromagnetic effects such as those provided by axions.

I. INTRODUCTION

Bartlett and Corle (BC) recently reported a measurement of the magnetic field associated with Maxwell's displacement current as it flows between the plates of a circular, parallel plate capacitor.[1] That measurement confirmed the classical prediction that there is an azimuthal B field that increases linearly with distance from the axis. The BC measurement, however, was only of the azimuthal component of the field; no measurement was made of the other two components. Such components could arise if, for instance, the displacement current twists in going from one plate to another, thus producing a B field in an unexpected direction. Although this possibility is hardly attractive, it is consistent with the concept of continuity that inspired Maxwell to write "Every case of charge or discharge may . . . be considered as a motion in a closed circuit, such that at every section of the circuit the same quantity of electricity crosses in the same time, and this is the case, not only in the voltaic circuit where it has always been recognized, but in those cases in which electricity has been generally supposed to be accumulated in certain places."[2]

This concept of current continuity may be expressed in differential form as

$$\nabla\cdot\mathbf{J}=\nabla\cdot\mathbf{J}_c+\nabla\cdot\mathbf{J}_D=-\partial\rho/\partial t+\nabla\cdot\mathbf{J}_D=0\ , \tag{1}$$

where $\mathbf{J}$, the total current density, is the sum of $\mathbf{J}_c$, the conduction current, and $\mathbf{J}_D$, the displacement current. But the continuity equation only specifies J_D up to the curl of $\mathbf{X}$ where $\mathbf{X}$ is an arbitrary vector field.[3,4] In choosing $\mathbf{J}_D=\partial\mathbf{D}/\partial t$, Maxwell effectively assumed $\mathbf{X}=0$. Although this choice is clearly the simplest one, experimental verification is necessary.

The present measurement differs from BC in using a coaxial rather than a circular capacitor. Imagine such a

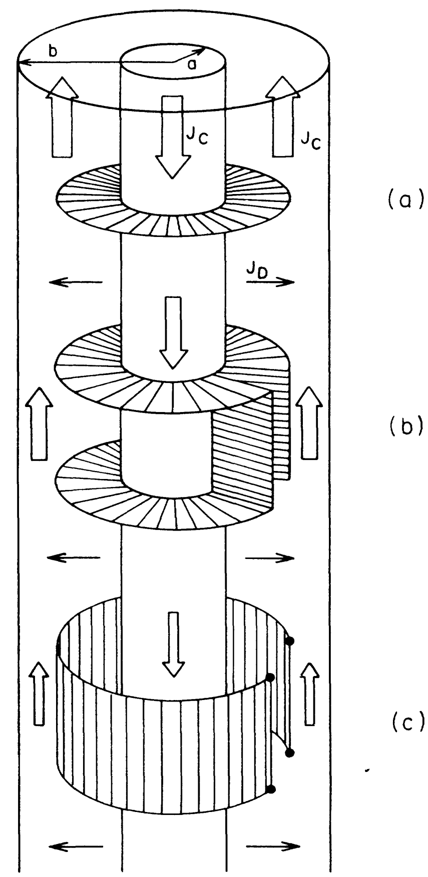

FIG. 1. Ampere's law and Maxwell's displacement current. The coaxial capacitor is fed from the top. (a) Surface used to establish $B_\phi=2I_c/c\rho$; (b) surface for $\Delta B_\phi=2\Delta I_c/c\rho$; (c) surface for $\Delta B_\phi=(1/c)(\partial D/\partial t)\Delta z$.

capacitor to be charged uniformly from the top with a steady current as indicated in Fig. 1. From elementary theory we predict the B field (cgs) inside the capacitor to be

$$B_\phi = \{z/[c\rho \ln(b/a)]\} dV/dt; \quad B_z = 0, \; B_\rho = 0 , \quad (2)$$

where a and b are the inner and outer radii of the capacitor.

A worrisome assumption in this conventional treatment is that the *conduction* current flowing in the cylinders has no ϕ component. In the analogous case of pure electrostatics it is indeed true that a symmetric geometry necessitates a symmetric charge distribution. There the requirement that $E=0$ inside the conductor plus the uniqueness theorem ensures the desired symmetry. For currents in ordinary metals, however, a symmetric geometry does not automatically generate a symmetric current distribution. Rather the currents are markedly influenced by such mundane features as internal voids, solder joints, asymmetric attachment points for feed electrodes and slight variations in the composition of the electrodes.

Fortunately superconductors do have the desired feature that a symmetric geometry implies a symmetric current distribution. This is because of the Meissner requirement that $\mathbf{B}=0$ inside the conductor.[5] When combined with the uniqueness theorem the Meissner requirement is sufficient to show that in a cylindrical geometry carrying both slowly varying charges and currents, the currents have no ϕ component. To see why this is so consider the situation illustrated in Fig. 2.

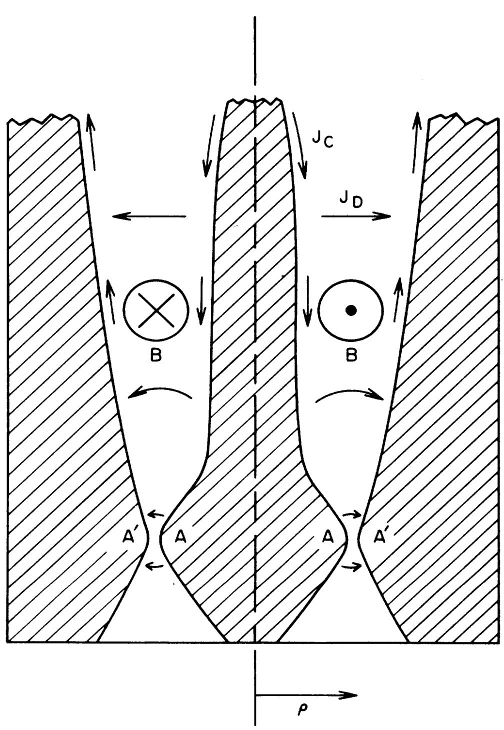

FIG. 2. Meissner effect and azimuthal symmetry. The B field has only a ϕ component. When surfaces A and A' touch, $J_D=0$ and $B_\phi = 2I/\rho c$.

A cylindrically symmetric distribution is clearly possible for the *charge* distribution. By the uniqueness theorem, this must be the solution. The displacement current $\mathbf{J}_D = (1/4\pi)(\partial \mathbf{D}/\partial t)$ is then also cylindrically symmetric. In a superconductor the conduction currents are superficial and are given by $\mathbf{K}_c = (c/4\pi)(\hat{\mathbf{n}} \times \mathbf{B})$. The solution with B having only a ϕ component is consistent with the geometry and thus possible. By the uniqueness theorem it is then the solution.

II. EXPERIMENT

As in the BC measurement we charge the capacitor at a low audio frequency (here 695 Hz) to simulate quasistatic conditions. We also use a superconducting quantum interference device (SQUID) to measure the small (mG) magnetic fields which the displacement current produces. In distinction to BC, however, we now measure all three components of the B field inside a cylindrical (rather than a parallel plate) capacitor. Since all access is

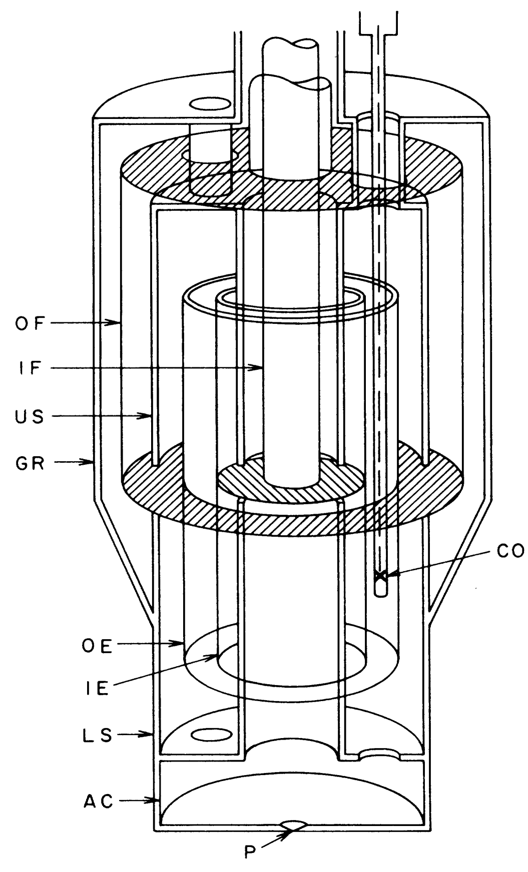

FIG. 3. Schematic of apparatus. Electrodes and feeds (shown in full): IE is the inner electrode; OE is the outer electrode; IF is the feed for inner electrode; OF is the feed for outer electrode. Shields (shown split): US is the upper shield; LS is the lower shield; GR is the ground return. At the bottom is the antechamber (AC) used for calibration. Electrodes form a capacitor having length $h=15.2$ cm, i.d. $2a=5.4$ cm, and o.d. $2b=7.6$ cm.

from the top of a liquid helium Dewar, the cylindrical geometry permits sampling the field at three different azimuths, thus providing a necessary consistency check.

The need to keep the grounded SQUID probe from altering either the electric or the magnetic field between the electrodes dictated the rather convoluted electrode and shield arrangement of Fig. 3. The outer and inner electrodes are driven coaxially at 108 V rms in the "push-pull" mode. An upper shield isolates the region between the electrodes from the electric fields surrounding the coaxial feed cylinders. A symmetric lower shield isolates the electrodes from stray electric fields. A small antechamber at the bottom can be used to enclose a toroid for calibration of the probe. Finally, all the electrodes, the electrode feeds, and the shields are fabricated from copper or brass tubing and flange stock and have been electroplated with 0.15 mm of lead. They are thus superconducting, and serve as magnetic shields.

At the top of the cryostat the three coaxial tubes feeding the electrodes and shields are diminished to a small radius. This "necking" permits the sensitive Josephson junction of the SQUID to be in a region of $\mathbf{E}=\mathbf{B}=0$; while leaving the SQUID's pick-up coil in between the electrodes. The coil is slanted at an angle of 70° with respect to the axis of the probe. Then, when the probe is rotated, the horizontal components of the field, B_ϕ and B_ρ, appear as the difference of measurements, whereas the vertical field B_z appears as the sum.

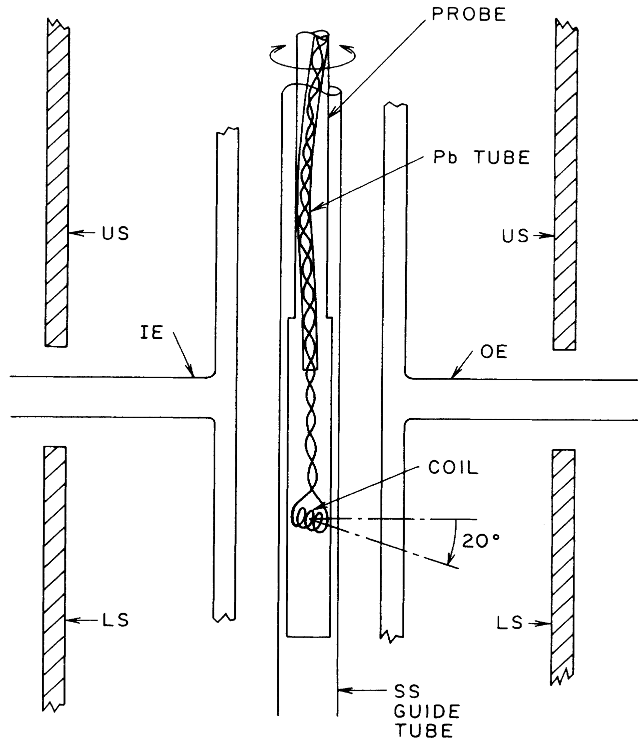

FIG. 4. Detail of pick-up coil.

A. SQUID Probe

The magnetic field is detected by a small coil coupled to a SQUID.[6] (See Fig. 4.) The coil consists of ten turns of 0.13-mm-diam Nb superconducting wire wrapped around a 1.5-mm-diam wooden peg. The peg fits across an insulated brass tube which is 3 mm in diameter and 0.2 mm thick. The brass tube slides and rotates inside a fixed stainless-steel (SS) tube which extends all the way through the antechamber (AC) used for calibration and has an o.d. of 4.6 mm and a thickness of 0.4 mm. (Note that this thickness is only 2.5% of a skin depth at a frequency of 695 Hz).[7] The three SS guide tubes are located midway between the electrodes. This feature keeps E induced longitudinal currents from flowing along the guide tubes, thus distorting the B field.

The SS guides are also midway azimuthally between three Teflon-coated Pyrex rods used to maintain very rigid control of the spacing between the electrodes. (See Fig. 5.) The even spacing was critical. Since Pyrex has a large dielectric constant, the rods increased the capacity (and hence B_ϕ) by almost 25%. Fortunately the horizontal polarization current in the rods does not produce spurious B_ρ or B_z fields in the symmetrically placed SS tubes.

B. Assembly

The capacitor sits inside a liquid-helium Dewar. (See Fig. 6.) Surrounding the Dewar is a highly permeable magnetic shield which reduces the Earth's field to 3 mG. Each electrode of the capacitor is driven by an oscillator coupled to an audio amplifier and a step-up transformer. A phase control ensures "push-pull" operation wherein the phase of the voltage on the inner electrode is opposed to that of the outer. In addition the oscillator supplies the reference signal for a "lock-in" amplifier used to detect the approximately 1-mV signals of the SQUID controller.

The height and the azimuthal angle of the coil are mea-

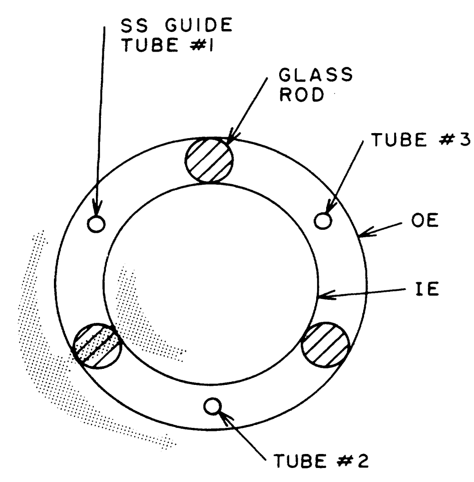

FIG. 5. Top view of Pyrex rods used to separate electrodes and SS tube to guide SQUID probe. Arrows show horizontal currents induced by polarization current in glass rod.

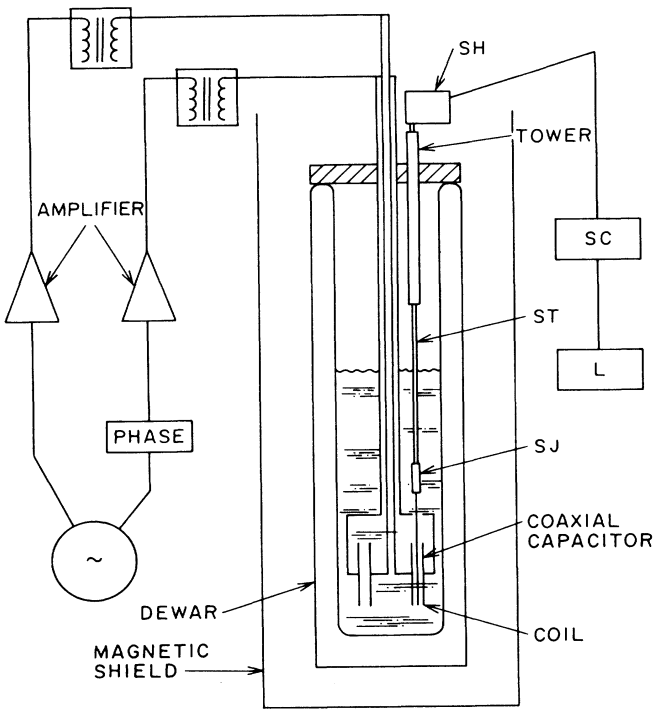

FIG. 6. Block diagram of complete experiment. SJ, ST, SH, and SC, are SQUID junction, transmission line, head, and controller. L is the lock-in amplifier.

sured at the top of the Dewar. Rigidly attached to the upper part of the transmission line of the SQUID is a stainless-steel tower. Scribed on the tower are horizontal bands spaced 1.27 cm apart and four vertical lines separated by $90\pm0.15°$. The height and angle of the coil are adjusted until an intersection of a vertical line and horizontal band is in contact with a fixed point on the head of the Dewar.

C. Unintentional twists

Early runs showed a positive effect. There appeared to be a radial B field B_ρ which increased with height. After some study we found that the cause for this was instrumental. We had inadvertently put small twists in our apparatus so that the apparently radial direction really was partly azimuthal.

One cause for a twist was easy to see. The SS tower for our radial and azimuthal measurements slides in a hole in the head of the Dewar which is some 80 cm above the cryostat. The benchmark indicating the radial direction is inscribed in the head of the Dewar. Thus there is an error if the hole in the head of the Dewar is rotated slightly from the corresponding hole in the cryostat.

We corrected for this rotation by an *in situ* calibration. We mounted a toroid in the antechamber (AC) at the bottom of the cryostat. The toroid was penetrated both by the Pyrex rods and the SS guide tubes. The former registered the toroid to the interelectrode geometry; the latter permitted us to insert the probe into a region of a calibrated field in a known azimuthal direction.

Although this procedure did calibrate the sensitivity of the pick-up coil, it did not completely remove the observed radial signal. This signal was now observed to increase quadratically with height. We then instituted an angular calibrating procedure that could establish the azimuthal direction magnetically at all heights, not just at the bottom of the cryostat. The procedure used the property of the Meissner effect that in a coaxial geometry carrying only *conduction* currents the magnetic field between the outer and inner electrodes is just $\mathbf{B}=(2I/c\rho)\hat{\boldsymbol{\phi}}$. (See Fig. 2.)

For angular calibration then we eliminated displacement currents completely by inserting a central rod all the way down the Dewar to a recess (P) at the bottom of the antechamber. Current I_c of frequency and magnitude similar to that of the displacement current in our regular run was fed into the rod and collected coaxially from the outer ground return (GR). As a consequence secondary currents were induced in all the electrodes and shields so that $\mathbf{B}=(2I_c/c\rho)\hat{\boldsymbol{\phi}}$ everywhere in the free space within GR and in particular in between the electrodes.

For all three SS guide tubes we found a linear dependence of the magnetically determined ϕ direction with height. The measured changes over the 24 cm from lower to upper shields were 8.0°, 7.2°, and 7.1° for SS tubes 1, 2, and 3, respectively. After the run we examined the cryostat directly. By placing a plane mirror across its bottom and sighting through holes for the SS tubes in the upper and lower shields we determined that the lower shield was indeed rotated by a measured 7.5° (clockwise) with respect to the top shield. Thus the SS guide tubes were not truly vertical but had a slight twist.

III. PROCEDURE

During the main data-taking run we alternated displacement current measurements with measurements using conduction currents only as described above. Both sets of measurements were taken for all three SS guide tubes. At each elevation four measurements were taken: R_+, A_+, R_-, and A_-, corresponding to the B field in the nominal radial (out and in) and azimuthal (counterclockwise and clockwise) directions.

In a final run we tested the response of our system to extreme forms of anomalous conditions. Specifically we measured the magnetic field as a function of height in one SS guide tube for three different situations: the asymmetric drive, the Q signal, and the second harmonic. In the first case, we applied voltage to only one electrode, the other being grounded. In the second, we subtracted 90° from the phase delay of the lock-in amplifier so as to be sensitive to signals in phase with the charge on the capacitor. In the third we doubled the frequency of the lock-in amplifier to 1390 Hz and measured both I_2 and Q_2 signals.

IV. RESULTS

We determine the B field by using the four measurements R_+, A_+, R_-, A_- in four equations:

$$B_\phi = (S/2)\{(A_+ - A_-)\cos[\theta(z)] - (R_+ - R_-)\sin[\theta(z)]\} ,$$

$$B_\rho = (S/2)\{(A_+ - A_-)\sin[\theta(z)] + (R_+ - R_-)\cos[\theta(z)]\} ,$$

$$B_z = (-S/4)\{(A_+ + A_- + R_+ + R_-)[\cot(20°) - Z]\} , \quad (3)$$

$$B_c = (S/4)(A_+ + A_- - R_+ - R_-) .$$

Here $S = 3.00$ μG/mV is the sensitivity of the pick-up coil; $\theta(z) = mz + b$ is the twist of the coordinate system as determined separately for each SS tube by using conduction currents only; and Z is the correction for a small drift in the zero of the lock-in amplifier. B_c is not really a field; rather it is a check on the internal consistency of the measurements. An isolated mismeasurement would be revealed as an anomalously large B_c.

Plots of B versus height z are given in Figs. 7(a)–7(d). In all figures we show data from the three azimuthally separated SS tubes. The figures show no evidence for departures from conventional theory. In Fig. 7(a) we show the expected field B_ϕ. The underlying curve is the theoretical expectation. Note that the curve does not go to zero at the bottom of the capacitor. This is because of displacement currents which flow from the inner electrode to the lower shield.

The curve in Fig. 7(a) was normalized to agree with the measured slope for the data

$$dB_\phi/dz(\text{meas}) = 0.121 \pm 0.004 \ \mu\text{G/cm} . \quad (4)$$

We compare this measured slope to the calculated slope,

$$dB\phi/dz(\text{calc}) = 2V\omega C/c\rho = 0.123 \pm 0.004 \ \mu\text{G/cm} . \quad (5)$$

In the calculation we use $V = 0.72$ sV $(2 \times 108$ V), $\omega = 4370$ rad/s, and $\rho = 3.26$ cm. After allowing for the liquid-helium dielectric, the basic capacity per unit length is

$$C = K/[2\ln(b/a)] = 1.05/[2\ln(3.8/2.7)] = 1.52 \text{ cm/cm (170 pF/m)} .$$

To this we add the experimentally determined augmentation in the capacity owing to the glass rods (0.32 cm/cm) and the SS guide tubes (0.07 cm/cm) to find a final C of 1.91 cm/cm.

Plots of B_ρ and B_z are shown in Figs. 7(b) and 7(c). There is no evidence for any significant signal. The average B_ρ signal in the region of the electrodes is 0.005 ± 0.004 μG or 0.3% of the maximum value of B_ϕ. For B_z the corresponding numbers are 0.014 ± 0.006 μG and 0.8%.

The plot of the check signal B_c [in Fig. 7(d)] does show a small but significant drift. The drift is -0.0016 μG/cm over the region of the electrodes. Since the check signal does not correspond to a real field, the drift must be instrumental. We do not know its cause but speculate that the center of the pick-up coil is not exactly centered in its housing. An eccentric coil will measure the azimu-

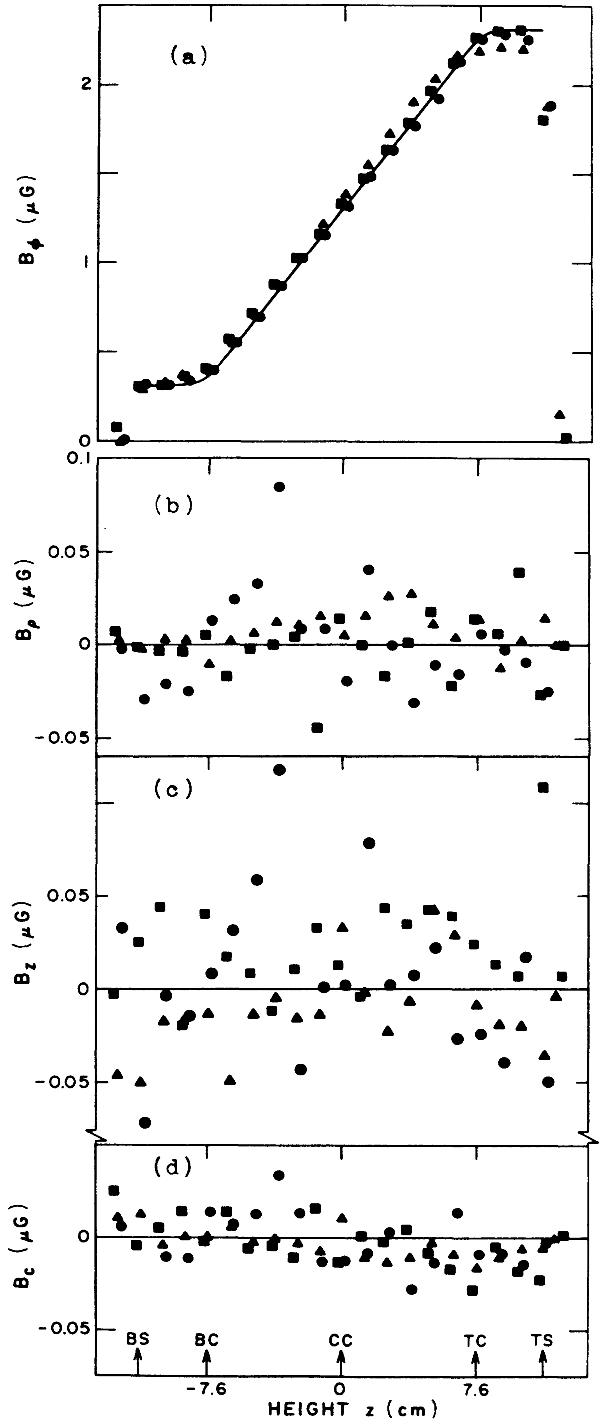

FIG. 7. (a) Results. B_ϕ vs z. BS is the bottom of lower shield, BC is the bottom of capacitor, CC is the center of capacitor, TC is the top of capacitor, TS is the top of upper shield. SS tube 1, triangles; SS tube 2, circles; SS tube 3, squares. Data were taken at same heights for all tubes, but for clarity circles and squares are presented with slight z offsets. (b) Results: B_ρ vs z. (c) Results: B_z vs z. (d) Results: check "field," B_c vs z.

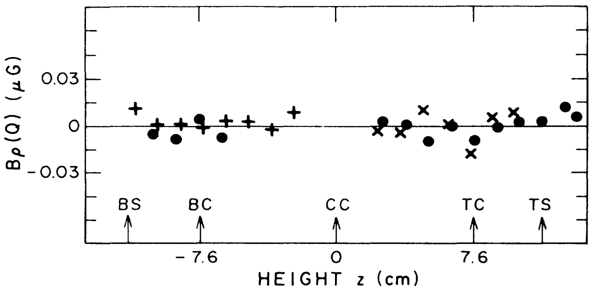

FIG. 8. Results: B_ρ (in phase with Q) vs z. Circles: "push-pull" voltages applied to both electrodes as in normal operation. Crosses: voltage only to outer electrode. Pluses: voltage only to inner electrode.

thal signal A_+ at a slightly different radius ρ than A_- and thus will give a B_c signal. A displacement between the centers of 0.8 mm would account for the signal.

As described above we made a final run to study anomalous conditions: asymmetric drive, Q signal, and second harmonics. The results were as expected. We showed that the B field with the normal push-pull drive is in fact the sum of the B field detected when the inner electrode is driven and the outer grounded (condition 1) and the B detected in the converse situation (condition 2). In neither condition 1 nor condition 2 did we observe any radial or longitudinal B field. Our limits on B_ρ and B_z are, respectively, 0.7% and 2% of 2.2 μG (the maximum observed B_ϕ).

We also failed to observe any B_ρ or B_z associated with the Q signal. (See Fig. 8.) We advanced the phase of the lock-in amplifier 90° so as to synchronize it with the charge rather than the current. We found that B_ρ and B_z were less than 0.3% and 3% of 2.2 μG, respectively (at all heights). We did observe a B_ϕ signal. This signal, which varied slowly over the duration of the run, was evidently caused by a drift of $\pm 5°$ in the phase delay of the lock-in amplifier.

Finally we doubled the frequency of the lock-in amplifier to 1390 Hz to detect signals which might vary as I^2, Q^2 or IQ. The first two would appear as a second-harmonic Q signal (Q_2); the last as a second harmonic I signal (I_2). We did not find any evidence for Q_2 or I_2 at any height. Our limits on the amplitude $S_2=(I_2^2+Q_2^2)^{1/2}$ are 0.07% and 0.6% of 2.2 μG for the ρ and z directions, respectively. For the ϕ direction we did find a signal of 2.4% of 2.2 μG. This signal, however, was consistent with that expected from the harmonic content of our driving field. The driving power as measured on a spectrum analyzer was 36 db lower at 1390 Hz than at 695 Hz.

V. DISCUSSION

It is clear that we have no evidence for an anomalous field, but what unusual B field could we expect? The most naive expectation is that the displacement current is not strictly radial but twists by an angle ϵ about the z axis. Thus we ignore end effects and suppose there is an anomalous current

$$\mathbf{J}_a=\epsilon|\mathbf{J}_D|\hat{\phi}=\epsilon\omega CV/(2\pi\rho)\hat{\phi} \tag{6}$$

in the region between the electrodes. This solenoidal current produces a B field midway between the electrodes of $B_z=4\pi K/c$ where

$$K=\int_{(a+b)/2}^{b} J_a\,dr=(\epsilon\omega CV/2\pi)\ln[2b/(a+b)] \ . \tag{7}$$

In our particular experiment, counter solenoidal currents will be induced in the Pb plated electrodes by the Meissner effect. A simple calculation shows that the induced currents reduce the expected B_z to $\frac{3}{4}$ of its unshielded value. Thus

$$\begin{aligned} B_z &=(3\epsilon\omega CV/2c)\ln[2b/(a+b)] \\ &=\tfrac{3}{8}[(b-a)/h]B_\phi(\max) \\ &=0.03B_\phi(\max) \ . \end{aligned} \tag{8}$$

Since the observed B_z is $<(0.008\pm0.003)B_\phi(\max)$, ϵ must be less than 22°.

The difficulty with this naive parametrization is that it is nonlocal, i.e., the axis for twist is determined by the geometry of the electrodes rather than by local fields. A local source for an anomalous current $\mathbf{J}_a=\nabla\times\mathbf{X}$ is provided by allowing $\mathbf{X}$ to be products of scalars or pseudoscalars and the fields $\mathbf{E}$ and $\mathbf{B}$ and their time derivatives.[8] Unfortunately none of the simplest contenders is attractive. For instance $\mathbf{X}$ could be proportional to B itself. But the effect of this would extend far beyond the present experiment. The magnetic field in *any* situation would be multiplied by a universal factor. Clearly this prospect is untenable.

The simplest choice for $\mathbf{X}$ that involves $\mathbf{E}$ and is consistent with time-reversal invariance is $\mathbf{X}=\text{const}\,\partial\mathbf{E}/\partial t$. (Here "const" is a pseudoscalar.) Faraday's law, however, ensures that the contribution of this term will vanish in the static limit:

$$\begin{aligned} J_a &=\nabla\times\mathbf{X}=\text{const}\,\partial(\nabla\times\mathbf{E})/\partial t \\ &=\text{const}(1/c)\partial^2\mathbf{B}/\partial t^2=0 \ . \end{aligned} \tag{9}$$

Thus our experiment would not be sensitive to this $\mathbf{X}$ even in the fringing field of the capacitor. (To be sensitive, one would want to measure the orientation of the magnetic fields associated with microwaves.[9])

The next simplest choices for $\mathbf{X}$ are nonlinear terms such as $\mathbf{U}\cdot\mathbf{U}\,\partial\mathbf{U}/\partial t$, where $\mathbf{U}$ is either $\mathbf{E}$ or $\mathbf{B}$. Here our experiment can set limits, but because of the weak fields in this experiment our limits are not competitive with those set elsewhere. An intriguing topical example of such nonlinear effects is provided by the axion.

Sikivie[10] has shown that the field a of the hypothetical pseudoscalar axion will modify Ampere's law,

$$\nabla\times\mathbf{B}-\partial\mathbf{E}/\partial t=K[\mathbf{E}\times\nabla a-\mathbf{B}(\partial a/\partial t)] \ . \tag{10}$$

Here $\hbar=c=1$ and $K=10^{-13}$ GeV^{-1} (0.2×10^{-26} cm) is roughly the quotient of e^2 and the inverse of the energy at which the Pecci-Quinn symmetry is broken. The field a satisfies the equation

$$(\nabla^2-\partial^2/\partial t^2)a=K\mathbf{E}\cdot\mathbf{B}-m^2 a \;, \qquad (11)$$

where $m\approx 10^{-3}$ eV (or 50 cm^{-1}).

In our quasistatic experiment the effect of the axion will be to give an anomalous current J_a which flows primarily in the region of the fringing field of the capacitor. To estimate the strength of this current, note that m is larger than the reciprocal of the extent of the fringing field, $1/\Delta z\approx 1/(b-a)\approx 1\ \mathrm{cm}^{-1}$. Thus the left-hand side of Eq. (11) may be neglected, giving $a=K\mathbf{E}\cdot\mathbf{B}/m^2$. Substituting this value for a in Eq. (11) we find that

$$J_a=K(\mathbf{E}\times\nabla a)\approx K^2E^2B/(m^2\Delta z)\;. \qquad (12)$$

The ratio of J_a to the normal displacement current $J_D=\omega E$ is very small,

$$J_a/J_D=K^2EB/(m^2\omega\Delta z)=10^{-35}\;. \qquad (13)$$

Here we have taken $B=3$ mG$=3\times 10^5\ \mathrm{cm}^{-2}$ to be the ambient static field in the dewar and $E=0.7$ sV/cm$=0.6\times 10^8\ \mathrm{cm}^{-2}$ to be the time-varying field between the electrodes. Since J_a has the time dependence of E^2, it would appear as a second harmonic (Q_2) signal (albeit one that is undetectable in our experiment).

In addition to the possibility of a twist in J_D, anomalous magnetic fields could arise if electric charge were a weak source of magnetic field (thus violating $\nabla\cdot\mathbf{B}=0$). Our experiment is not sensitive to such a source. This is because the charges on the electrodes are shielded from the pickup coil by the charges induced on the SS guide tubes. Fortunately another experiment has already ruled out the possibility of electric charge as a source for magnetic field. Vant-Hull has found that the magnetic monopole moments of the electron, proton, and neutron are all less than 4×10^{-24} Dirac monopoles.[11]

In contrast to our insensitivity to electric charge as a source for magnetic field, we are sensitive to electric *field* as a possible source. Specifically consider that the observed E and B fields in the capacitor result from the superposition of two traveling TEM waves: an incident downward wave and a reflected upward one. By Maxwell's theory for each traveling wave the radial E field, azimuthal B field, and direction of propagation form an orthongonal triad. Further, on reflection from the open end of the capacitor the two traveling E fields add and the B fields cancel. If $\omega h/c\ll 1$, the resultant standing TEM wave consists of a radial E field $E_\rho=E_0\cos(\omega t)$ and a small azimuthal B field, $B_\phi=-(\omega z/c)E_0\sin(\omega t)$, as observed.

Now suppose that, owing to some non-Maxwellian effect,[12] the E and B fields of the original traveling waves are not strictly perpendicular, but that the traveling B field has a small radial component, $B_\rho=\epsilon E_\rho$. Assume isotropy of space. The reflected and incident traveling waves will then have radial B fields which add to produce a standing field $B_\rho=\epsilon E_0\cos(\omega t)$.

From our measurement of the radial Q signal (Fig. 9) we can set a limit

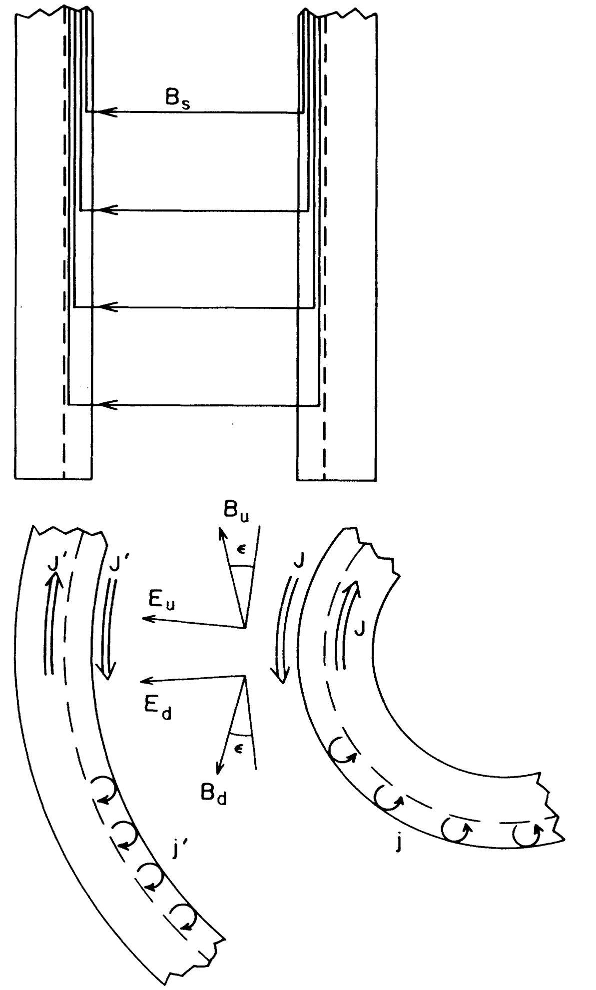

FIG. 9. Standing radial B_ρ generated from downward (E_d,B_d) and upward (E_u,B_u) TEM waves, each with a component of B parallel to E. Currents J and J' prevent penetration of B into superconductor. Corresponding Amperian loops j and j' are classical analogues of Dirac strings which end uniformly along the electrode.

$$\epsilon<B_{\rho 0}/E_0=0.007\ \mu\mathrm{G}/(0.76\ \mathrm{sV}/1.1\ \mathrm{cm})=0.8\times 10^{-8}\;. \qquad (14)$$

It may appear that a radial B field would necessarily cause problems at the electrodes. Either the field stops, producing a magnetic monopole, or it penetrates, thus violating the Meissner effect. In fact, an intermediate situation is possible. The field bends abruptly upwards within the Landau penetration depth. (See Fig. 9.)

ACKNOWLEDGMENTS

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*Present address: Sheldahl Corp., Northfield, Minnesota.

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