CODEN:LUTEDX/(TEAT-5027)/1-55/(1996)

Longitudinal electrodynamic forces — and their possible technological applications

Master of Science Thesis
by
Lars Johansson

Thesis Adviser: Professor Gerhard Kristensson

Department of Electromagnetic Theory
Lund Institute of Technology
P.O. Box 118
S-221 00 Lund, SWEDEN

Lund, September 18, 1996

Abstract

This thesis deals with the relatively unknown phenomena of longitudinal forces in conductors. A survey of relevant experiments is given. Ampère electrodynamics, which has been proposed to account for the phenomena, is compared with a Maxwell stress approach, and these are found to be equivalent. The relationship between different 'action at a distance' theories is discussed. The role of relativistic electric fields, and electric fields from surface charges, is considered.

Several interesting applications such as metal punching, liquid metal pumping, water-arc jet propulsion, high current limiting and electrodynamic plasma fusion are covered, together with a novel application: the electrodynamic explosion motor.

Contents

Chapter 1

Introduction

During the last decade a discussion on longitudinal forces in conductors has been going on. The discussion is not new though — it has recurred since the time of Ampère. In order to gain perspective a historical review is given in this chapter. In Chapter 2, I present some of the experiments connected with the phenomena. Various theories have been proposed to account for or disprove the existence of longitudinal forces. These are discussed in Chapter 3. Out of this discussion methods for calculation emerges, and the experiments are reexamined in the light of this in Chapter 4. The preceding discussion raises the question of possible applications of these phenomena. Some interesting applications such as metal punching, water-jet propulsion, high-current limiting and electrodynamic fusion are covered in Chapter 5. A novel application, the electrodynamic explosion motor, is also presented. The results are summarized in Chapter 6, followed by an extensive list of references.

1.1 Objective

The aim of this thesis is to awaken interest in the relatively unknown phenomenon of longitudinal forces in conductors. A survey of the area and some of its applications are given, together with a thorough theoretical discussion.

1.2 Acknowledgement

I would especially like to thank Dr. Peter Graneau and Dr. Jan Nasiłowski, for supplying me with articles and photos, and for answering my many questions.

1.3 Background and historical review

The idea of longitudinal forces in electrodynamics (forces in the direction of current flow) dates back to the time of Ampère. In order to calculate the magnetic forces between two electric currents, Ampère subdivided each circuit into 'current elements', Figure 1.1A and B. Through a series of ingenious experiments and some mathematical assumptions he arrived with a formula for the force between two such current elements. The total force on one of the circuits was then calculated by summing all the interactions between current elements in the different circuits.

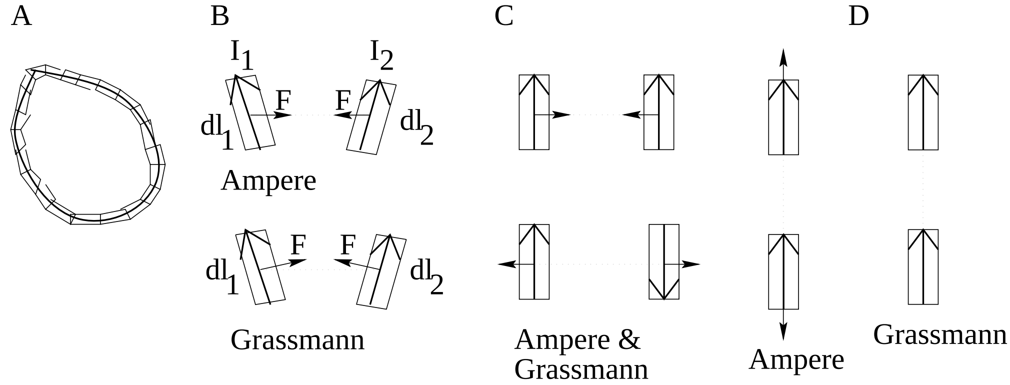

Figure 1.1: (A) Conductor subdivided into current elements. The sum of the forces from the current elements in one circuit on those in another gives the total magnetic force on that circuit. (B) Forces between current elements. (C) Transverse forces between parallel elements. (D) Longitudinal forces between co-linear elements.

$$d\mathbf{F}_{Amp} = \frac{\mu_0 I_1 I_2}{4\pi r_{12}^2}[3(d\mathbf{l}_1 \cdot \hat{\mathbf{r}}_{12})(d\mathbf{l}_2 \cdot \hat{\mathbf{r}}_{12}) - 2(d\mathbf{l}_1 \cdot d\mathbf{l}_2)]\hat{\mathbf{r}}_{12}$$

Here[1] $d\mathbf{F}$ is the force the current element 1 ($d\mathbf{l}_1$) exerts on current element 2 ($d\mathbf{l}_2$); I_1 and I_2 are the electric currents in the circuits; μ_0 is the magnetic permeability (a conversion factor); $d\mathbf{l}_1$ and $d\mathbf{l}_2$ are vectors in the directions of the current elements, see Figure 1.1B; r_{12} is the distance between the current elements; and $\hat{\mathbf{r}}_{\mathbf{12}}$ the unit vector in the direction from $d\mathbf{l}_1$ to $d\mathbf{l}_2$.

This once famous formula, now almost forgotten in textbooks, predicted a longitudinal repulsion between two co-linear current elements, Figure 1.1D. To test the theory, Ampère and de la Rive performed an experiment in 1822, where a hairpin was to be propelled along two troughs of liquid mercury by the longitudinal repulsion [7]. It did so, and the experiment was considered a success for Ampère's theory.

[1] See the appendix for a description of the notation.

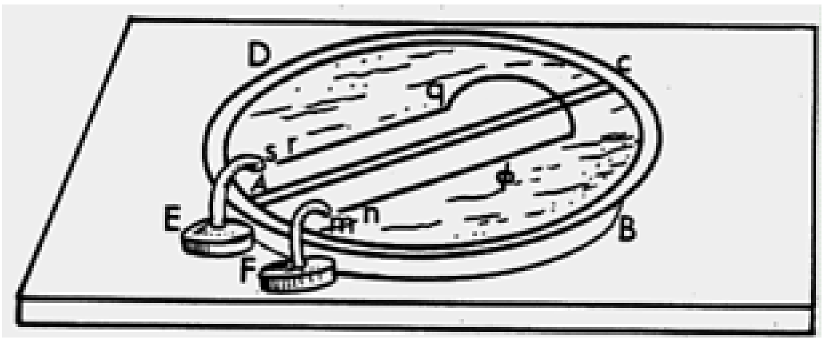

Figure 1.2: The hairpin experiment by Ampère and de la Rive.

Longitudinal forces were not predicted by the force law derived by Grassmann.

$$d\mathbf{F}_{Grass} = \frac{\mu_0 I_1 I_2}{4\pi r_{12}^2}[d\mathbf{l}_2 \times (d\mathbf{l}_1 \times \hat{\mathbf{r}}_{12})] = \frac{\mu_0 I_1 I_2}{4\pi r_{12}^2}[(d\mathbf{l}_2 \cdot \hat{\mathbf{r}}_{12})d\mathbf{l}_1 - (d\mathbf{l}_1 \cdot d\mathbf{l}_2)\hat{\mathbf{r}}_{12}]$$

His formula is sometimes incorrectly referred to as Ampère's force law in modern textbooks [39]. It can be derived from Biot-Savart's law and the Lorentz force[2]. According to Grassmann's law all the forces would reside in the transverse part of the hairpin, and thus only pull it from the front, rather than also push the ends from behind, as predicted by Ampère. A discussion arose regarding which law was the correct one [65]. When integrated around a circuit they both yielded the same result, the Neumann force law for two circuits [17, 9].

$$\mathbf{F}_{Neu} = -\frac{\mu_0 I_1 I_2}{4\pi} \oint\oint \frac{d\mathbf{l}_1 \cdot d\mathbf{l}_2}{r_{12}^2}\hat{\mathbf{r}}_{12}$$

But what about two parts of the same circuit? Ampère's followers advocated the hairpin experiment favored Ampère's law, but e.g. Maxwell argued that no matter what law was used the total force on the hairpin would be the same, as the laws were equal when a whole circuit was considered [43]. The possibility to measure exactly where in the circuit the forces resided was a question not pondered upon. Maxwell though remarked [44]:

> 'Of these four different assumptions [force laws] that of Ampère is undoubtedly the best, since it is the only one which makes the forces on the two elements not only equal and opposite but in the straight line which joins them.'

That is, only Ampère's formula obeys Newton's third law of equal action and reaction *locally*. The fact that Grassmann's (and thus Lorentz') doesn't was essential for the development of special relativity.

[2]See the appendix.

As the continental 'action at a distance' theories gave way to Maxwell's field theory, Ampère's law was abandoned. Grassmann's law fitted in better with Maxwell's theory, as it was easy to relate his force to the magnetic field. Longitudinal forces were passed into the shadows for a while.

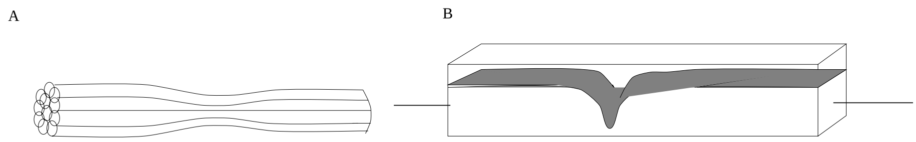

Figure 1.3: The pinch effect. (A) As 'like currents attract', the conductor squeezes itself radially inward. If the material is too weak to sustain the pressure it is pinched off at some point. (B) Pinch depression in a liquid conductor, caused by a current of some hundred Amperes.

The debate rekindled in 1921, initiated by Carl Hering who in 1907 had discovered the pinch effect in liquid conductors, Figure 1.3. He had also observed longitudinal forces but these had up to then been neglected by others.

In the 20s, confusion still reigned as to what laws of magnetic forces to be used. Engineers tended to rely on various laws of induction and reasonings like 'like currents attract, unlike repel', i.e. Neumann's law for two different circuits. Hering wrote several papers on the revision of these views, which spurred debate. His views on the matters were expressed clearly at the A.I.E.E. Midwinter convention in 1923:

> 'The strenuously opposed and long neglected longitudinal force was strongly upheld by some able discussers and disproved by none; the worst that the opponents could say about it was to the effect that they could get along without it.'

The fact that Maxwell's own field theory at least partially accounted for the longitudinal forces was clearly recognized by Hering. In Maxwell's theory a tension along the lines of magnetic and electric field existed, together with a pressure between the lines. Faraday had visualized lines of force and explained his discoveries in this geometrical language. Maxwell had cast Faraday's concepts into mathematical form and concluded that these stresses must exist in a medium — the ether. From an hydrodynamical analogy he had then derived the existence of electromagnetic waves.

With special relativity the ether was abandoned — although in a sense it wasn't, as Maxwell's stresses survived into Minkowski's energy-momentum-stress tensor. Minkowski's tensor was crucial for Einstein's development of general relativity [12], but is seldom covered to any extent in modern introductory textbooks — they tend to discuss forces with the aid of the Lorentz equation.

In the 50s, Moon and Spencer derived a field free electrodynamics from Ampère's formula, as an alternative to field theory. They pointed out that induction and magnetic forces are by no means unambiguous from a student's point of view, if you go beyond textbook examples. They also noted that the debate around these paradoxes showed a certain pattern: '... interest seems to rise anew to a sharp peak, after which it gradually declines, leaving the subject much as it were before.' [48].

This was the state of affairs when Peter Graneau rediscovered some of Hering's experiments in the beginning of the 80s and begun a search for longitudinal forces in literature and experiments [27]. He discovered the quite unknown papers by Jan Nasiłowski, a Polish scientist who had discovered electrodynamic wire fragmentation in the 60s. A new debate begun, adding to the above-mentioned phenomena anomalous railgun bucking, electrodynamic explosions in water and several other things.

Chapter 2

Experiments

In this chapter we discuss some (quite unnoticed) experiments in which longitudinal forces reveal themselves. As we shall see, several of the experiments are open to different interpretations. The discussion hopefully sheds some light on where we could expect longitudinal forces in experiments. The presentation naturally has the character of a survey — I refer to the references for details[1].

2.1 Longitudinal forces in solids

Nasiłowski's wire fragmentation

In 1961, Jan Nasiłowski performed experiments with electrodynamic wire explosions [51, 52]. When subjected to a current pulse of sufficient magnitude a thin

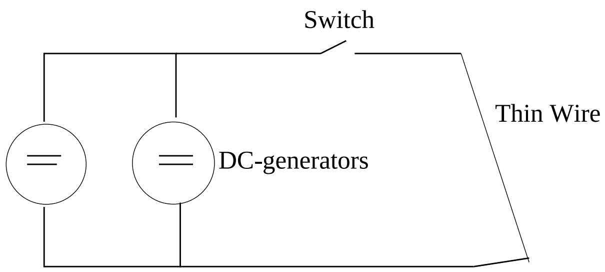

Figure 2.1: Nasiłowski's circuit for the creation of wire fragments.

wire disintegrated into pieces in solid state, Figure 2.2. When the segments were investigated it was found that the breaks were due tensile stress. Nasiłowski found

[1] Ref. [17] covers most of the experiments up to 1985. Many of the liquid metal experiments are found in Ref. [35]. The plasma experiments and an update on the whole area will be covered in Ref. [28], according to Peter Graneau.

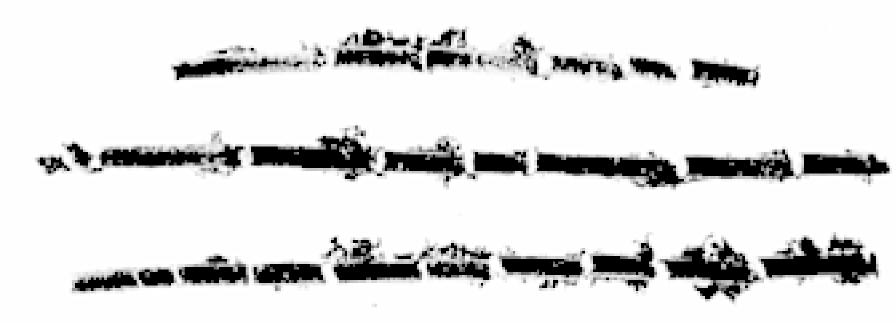

Figure 2.2: Wire fragments created by a wire explosion. From Jan Nasiłowski, Instytut Elektrotechniki, Warsaw, Poland.

that a minimum current was needed to shatter the wire. By using wires of different diameters and materials, he derived that the tensile force depended on the square of the current. However, the wires shattered after the current peak. Thus it wasn't until the conductor was severely weakened by heating that the breaks could occur — typically when the wire temperature had risen to 800-950 $^\circ$C.

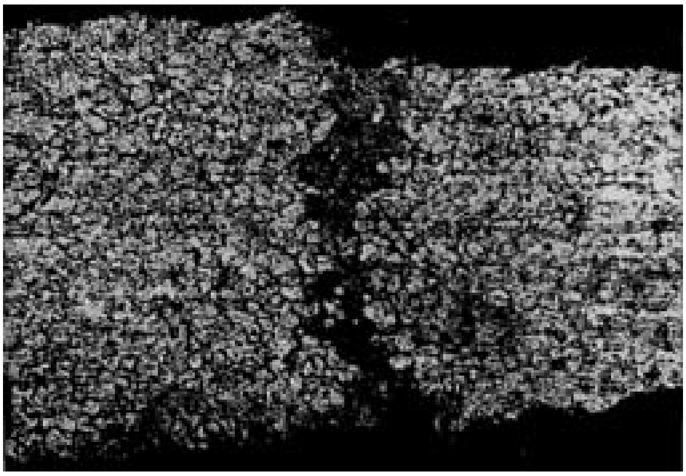

Figure 2.3: Cracks due to stress in the wire. From Jan Nasiłowski, Instytut Elektrotechniki, Warsaw, Poland.

A section of a wire crack is shown in Figure 2.3. The dark band is molten metal created by the arcing in the gap.

Formerly it was believed that all wire disintegrations were due to local wire melting, as unduloids formed during overloading of a wire [4], Figure 2.4. Nasiłowski showed that wire fragmentation could be caused independently of the unduloid formation.

The experiments were repeated by Graneau with a slightly different setup, Figure 2.5. In order to avoid melting of the wire surface the DC-generators were substituted by a high voltage capacitor bank. Short high current pulses should be able

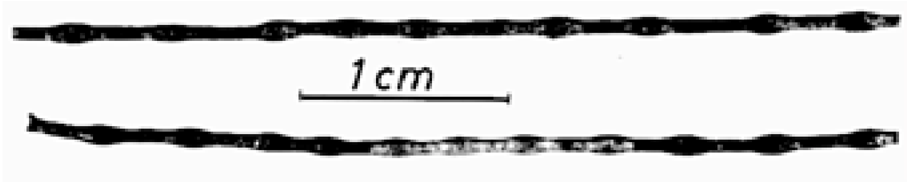

Figure 2.4: Unduloids formed by surface melting and pinch pressure. From Jan Nasiłowski, Instytut Elektrotechniki, Warsaw, Poland.

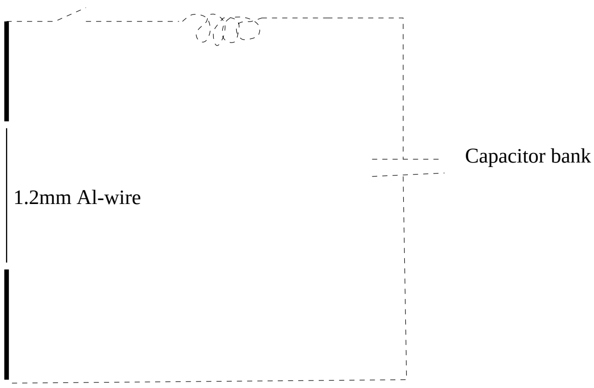

Figure 2.5: Graneau's experimental setup for wire fragmentation.

to break the conductor before heat had been absorbed to such a degree as to melt the surface. This was exactly what Graneau found [17, 14, 16].

When the 1.2mm Al-wire was subjected to a short current pulse in the range 5-7kA, it broke into two pieces. With a higher current successive breaks occurred. Electron microscope investigation of the surfaces showed that the breaks were due to tensile stress.

Different explanations of the phenomenon have been proposed:

- Pinch forces would create a uniform stress in the radial direction. This undoubtedly caused the unduloids. But the wires that shattered showed no pinch off. However, as noted in some references, a conductor could break due to a uniform radial force — causing a rather clean break [2, 8]. The force though seems to be too small to alone cause such a break.

- Accumulating thermal stress waves were proposed by Ternan [64], to explain Graneau's experiments at MIT where the ends were free to move. This explanation was discarded as it could not explain Nasiłowski's experiment where the ends were clamped. Later experiments showed that the wire could break in any state [20]. This is also evident from some of Nasiłowski's striation photographs [51]. However, Nasiłowski detected vibrations during the explosion [52]. If a force could cause the wires to break, stress waves due to the rupture would travel in the segments — causing standing stress waves. This may be a reason why the smaller segments show a reasonably regular spacing.

- Longitudinal forces would according to Ampère's formula set up a stress in the wire in addition to the pinch pressure. According to Graneau, this stress would be substantially greater than the pinch pressure.

The multi-arc generator

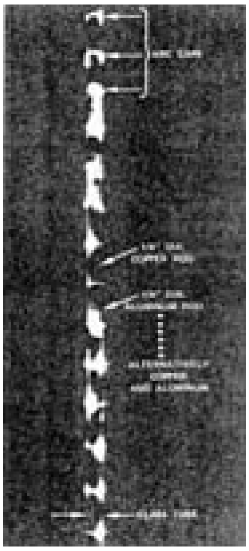

Figure 2.6: Separation and arc formation in the multi-arc generator. From Peter Graneau, North-Eastern University, Boston, USA.

In Nasiłowski's experiment a longitudinal stress, or repulsion, was observed in the interior of a conductor. The question now arose if this repulsion could be observed more directly. This was studied by Ruscak and Bruce [62]. A 6.4mm copper rod was cut into 1cm pieces and stacked to 1m length in a glass tube in vertical position. A light spring kept the pieces close together. When current pulses of 3-30kA were discharged through the column the pieces separated and arcs formed in between them, Figure 2.6. Some pieces spotwelded together by the arcs. Clearly, the rod pieces seemed to repel each other.

Railgun recoil

Railguns were of interest in the 80s SDI-program. Today, interest in electromagnetic launching has revived in NASA's plans for a new kind of a space-shuttle. A railgun consists of two parallel bars and a transverse rod, the sleigh, Figure 2.7A. When a current is passed through the circuit the sleigh is accelerated due to the Lorentz forces (the conduction electrons in the sleigh move in the magnetic field from the bars). As can be seen, Figure 2.7B, these forces are strongest in the corner regions, and directed at right angles to the conductor, i.e. they are transverse. The recoil forces were expected to be seated in the rear, section III in Figure 2.7B. But recoil forces were also observed in the rails, the arrows in Figure 2.7A, pushing the rails back and thus deforming them. Some experimenters observed plastic deformation of the rails [5]. Others reported severe friction losses, which could be generated by transient bucking of the rails [21, 55].

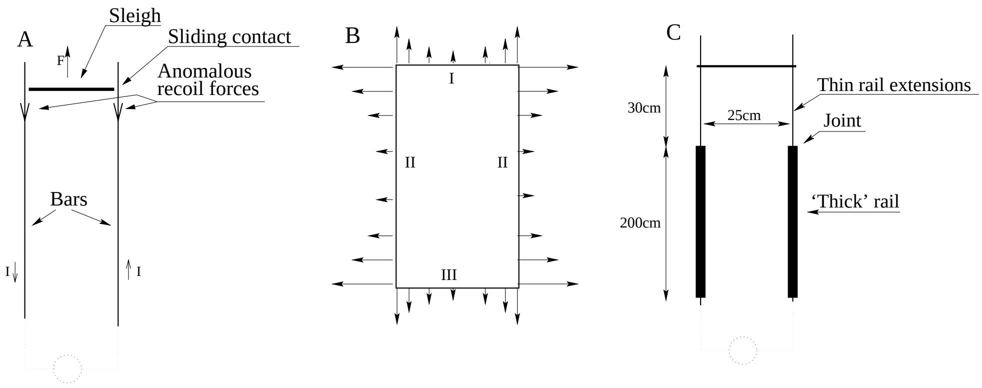

Figure 2.7: (A) Railgun principle. (B) Lorentz forces in the rectangular Railgun circuit. (C) Graneau's experimental setup.

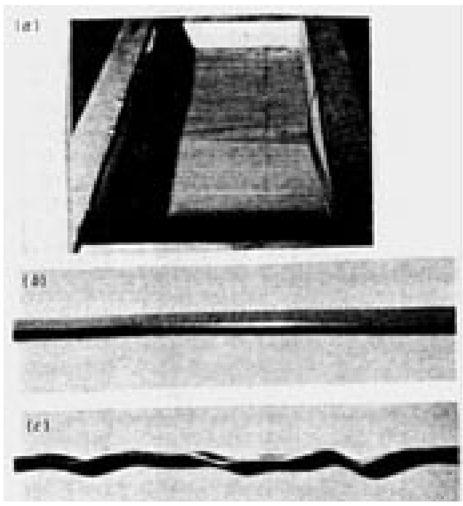

Figure 2.8: (A) Inward deformation of aluminium rails. (B) Rails before deformation. (C) Bucking of steel rails. From Peter Graneau, North-Eastern University, Boston, USA.

Peter Graneau performed an experiment with thin rail extensions, Figure 2.7C, to demonstrate the seat of the recoil forces [21]. As the extensions were thin, they would easily be deformed if subjected to a backward push (like Euler breaking of a rod). The transverse sleigh was held fixed. When current pulses of up to 100kA were applied severe deformation of the rail extensions was observed. Figure 2.8 shows some of the deformed rails. Longitudinal recoil forces were obviously present in the rails (and rail extensions), deforming the parts that were unable to sustain the compression.

Measurement of repulsion

The possibility of actually weighing the repulsion between different parts of a circuit was first investigated by Cleveland [10]. Since then, many experiments have been performed to measure the force that one part of a conductor exerts on another part.

An experiment that often have been used in the debate is the impulse pendulum[2], invented by Pappas. It consists of a horizontally suspended rectangular frame cut in two, where one of the halves is free to move. A current pulse imparts momentum on the moving pi-frame, which is carefully measured. Precise measurements have been made by Moyssides [50] and Peoglos [54]. The total force seems to be correctly given by the Lorentz force. However, the Lorentz law does not account for the the fact that the intermediate parts of the circuit are in a state of stress.

2.2 Longitudinal forces in liquid metals

Let us now study forces in liquid metal conductors. An abundance of experiments exists in this area, performed by Ampère [7], Neumann, Hering [35], Graneau [17] and others. Some of the most illuminating will be presented here.

Ampère's hairpin experiment

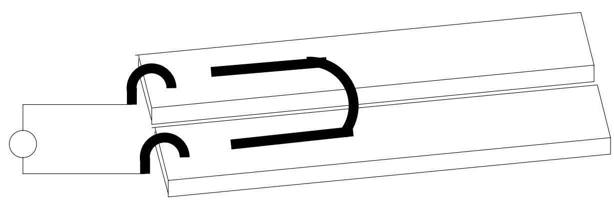

Figure 2.9: One version of the hairpin experiment.

In this experiment, first performed by Ampère and de la Rive in 1822 [7], a bent metallic conductor is floating along two troughs filled with mercury when a current

[2]Most of the references to this debate are found in Ref. [30].

is passed through the circuit. The experiment was repeated by Tait, who substituted the hairpin with a mercury filled siphon [63]. This ruled out the possibility of thermal forces at the copper-mercury interfaces, which had been proposed as an alternative explanation. As mentioned above, Ampère's law would predict a force on the legs of the hairpin, whereas Grassmann's and Lorentz' would have all of the force seated in the transverse segment.

The hairpin experiment was repeated by Graneau [13], who noted that mercury was repelled from the rear faces of the hairpin. Ampère's law would explain this as a result of longitudinal repulsion, whereas field theory would interpret it as an effect of the diverging current in the mercury, just outside the face of the copper hairpin [36].

Agitation and wave formation

In order to investigate the mercury repulsion further, the following experiment was performed [17]: In a long rectangular mercury trough, with copper bars at its ends,

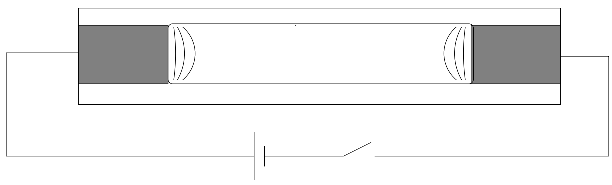

Figure 2.10: Agitation at the interface between solid and liquid metal.

a current of some hundred Ampère was passed. Wave patterns were observed near the the copper faces indicating a non-equilibrium in the conductor. Liquid metal seemed to flow away from the center of the copper face and then recirculate back at the periphery. The conductor thus seems to stretch itself at its center, like a compressed spring.

This is illustrated quite well in the liquid mercury fountain experiment in Figure 2.11:

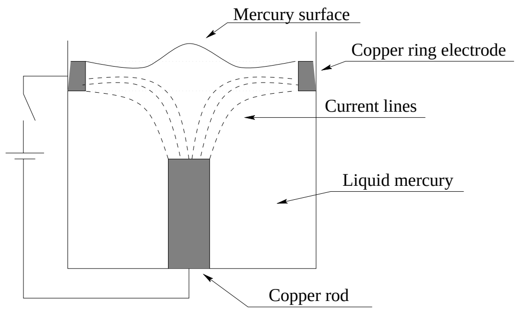

Figure 2.11: The liquid mercury fountain.

Metal flows away from the rod in the middle as it is being repelled, and the recirculates back. The jet from the rod causes the surface to rise in the middle of the cup [23].

Movement due to asymmetry

One debated experiment is the copper submarine, invented by Northrup [53]. A pointed rod is floating in a mercury trough. When a current of approximately 400A is passed through the trough, the rod submerges and is propelled along the trough with the blunt end first, Figure 2.12. The submersion is due to the pinch forces (as 'like currents attract').

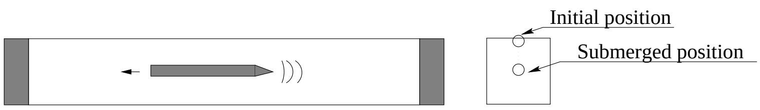

Figure 2.12: The copper submarine.

The cause of the longitudinal motion is more debated. Northrup [53] attributed this to the hydrostatical pressure in the center, which is caused by the pinch forces. Hillas [36] has interpreted it as caused by the diverging currents at its ends. Graneau [13] suggested that the longitudinal repulsion predicted by Ampère's formula to be the cause of the movement.

The direction is the same if the current is reversed or if alternating current is used. This seems to be common for all these experiments, the forces varying with the square of the current.

In one of Hering's experiments a hooked wire is dipping into two mercury cups, one being narrow and the other one a wide dish,Figure 2.13. The wire moves decidedly to the right when a current is passed through. Hering explained this as being due to the stretching of the magnetic flux (the circles of magnetic flux around a conductor repelling each other). As the current is more concentrated in the narrow trough than in the wide dish (where the current diverges), the magnetic repulsion from the trough dominates, pushing the suspended wire to the right. The movement is independent of the direction of the current.

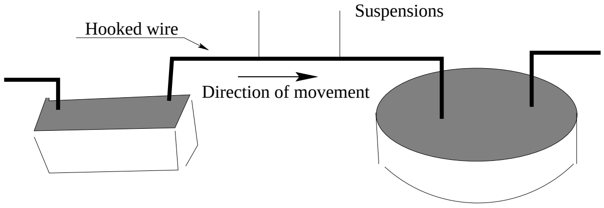

Figure 2.13: Wire movement between a narrow trough and a wide dish.

Conductor stretching

In order to demonstrate that a conductor tends to stretch itself, Hering devised several experiments [35]. In one experiment a chain of copper was suspended in a mercury trough, Figure 2.14. When current was passed through the circuit the

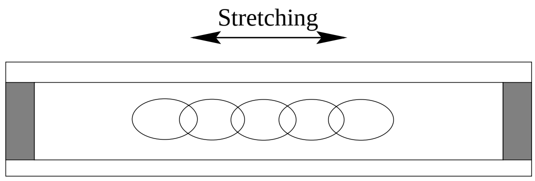

Figure 2.14: Stretching of a copper chain.

chain stretched itself to its full length. Hering attributed this to the presence of more concentrated flux around the copper chain than around the mercury at its ends, the flux consequently trying to stretch the chain. He wrote:

> 'This experiment also meets the claims sometimes made that in some of the movements in these experiments it is the pinch effect which by its hydrodynamic action in the mercury causes the motion. If the pinch effect were to cause, it would act to push the ends of the chain towards the middle, but the fact is that the movement is in the *opposite* direction.'

A variant of this experiment was made by Graneau, where the chain was substituted by two copper bars, originally in contact, which separated under the action of the current, Figure 2.15. The repulsion between the bars was thus stronger than the repulsion between the copper-mercury interface.

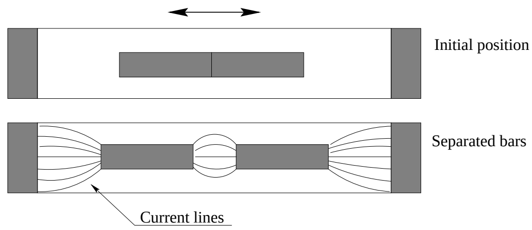

Figure 2.15: Repulsion between two copper rods originally in contact with each other.

2.3 Longitudinal forces in dense plasmas

In the previous sections we have studied forces in solids and liquid metals. Could the discussion be extended to dense plasmas, such as ionized water? As we shall see, new phenomena arise, making the situation a bit more complicated.

Electrodynamic explosions in water

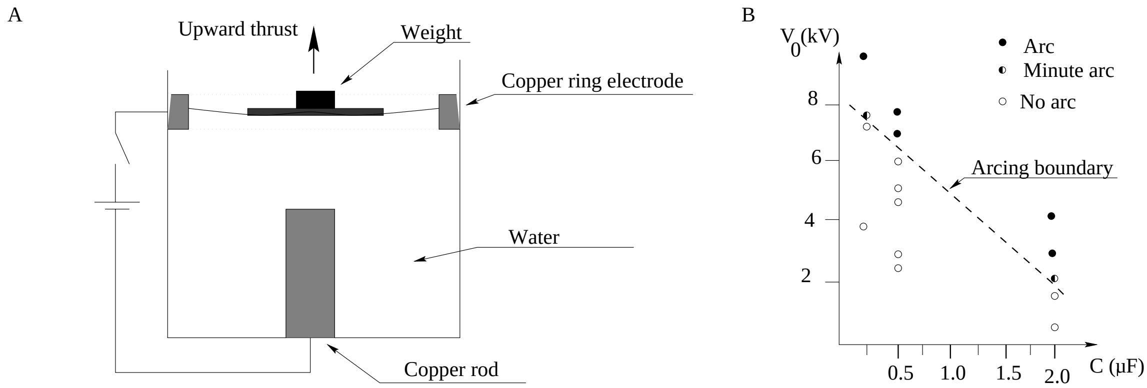

Figure 2.16: (A)The modified fountain experiment. (B) Explosion threshold.

Peter and Neal Graneau carried out an experiment to look for longitudinal forces in water [18], Figure 2.16A. The mercury in the fountain experiment was replaced with water of varying salinity. Current pulses were discharged through the cup. In some cases the currents discharged silently, in others a luminous arc struck between the rod and the ring, accompanied by a hissing sound and shock waves in the water. The boundary between the two kinds of discharge depended on the total charge (Q) in the discharge, Figure 2.16B. This suggested that an arc formed when the number of ions present in the solution was insufficient to discharge the capacitor. The energy not dissipated through the electrolytic current obviously went into the arc discharge. With the same amount of energy, the discharge could either be silent or cause an arc explosion, depending on the combinations of the capacity and voltage of the capacitor.

In a typical experiment the average force during the explosion was 21.6N, throwing a 2.8g weight that was floating on the surface about 20cm up in the air, the current being about 94A. The calculated thrust from pinch forces was 0.55mN, too small to account for the force. Pinch thrust would also be present during the silent explosions, which proves that the observed force has something to do with the presence of an arc. With another arrangement [19], Figure 2.17, current pulses in the range 10-

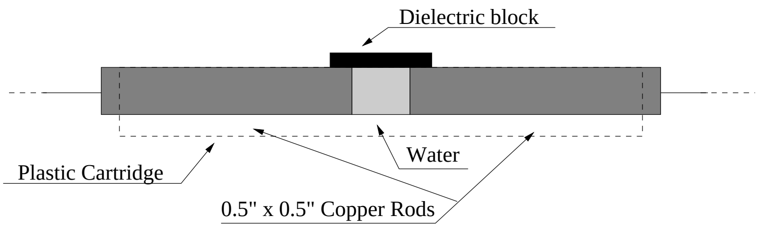

Figure 2.17: Experiment to create powerful water-plasma explosions.

25kA were used. The maximum force observed amounted to 430kN, equivalent to a pressure of 27'000 atm. If the water was free to move it would be expelled, emitting intense white light. When captured, it was found to be only lukewarm.

What is the cause of the explosions ?

- Thermal forces due to Joule heating of the water seem unlikely, as the same amount of energy dissipated could cause explosions in one case, and discharge silently in another. The energy needed for thermal explosions has been debated [11, 25]. A related subject is the cause of thunder. Peter Graneau has suggested that thunder may be driven by electrodynamic explosions, and not by thermic heating of the air [22]. An interesting question would be if the water content in the air affects the intensity of the thunder.

- Superheated steam has been suggested to drive a thermal explosion. The energy available may be too small for this to happen, as the discharge then would have to be confined to a small filament in the water. The latter is contradicted by the fact that the arc tends to fill the whole gap [25]. More important though is that no steam has been detected in any experiment, although a great deal of the water can convert into cold fog and mist.

- The production of fog and mist has made Peter and Neal Graneau suggest that the chemical bonding energy may be altered. Recent measurements also indicate that the energy released substantially exceeds the energy supplied by the arc [28, 29]. Their theory is that the bonding energy of very small droplets may be less (i.e. more negative) than that of liquid water. The difference in energy would then be released during the conversion into fog. Investigations indicate that only the smallest droplets explode, a fact supporting the theory.

If the excess energy turns out to be real this would be an interesting kind of 'solar energy' (as it is the solar radiation that has to do the endothermic conversion from mist to steam), the Graneaus point out. Clearly, the phenomenon of water-arc explosions is still not well understood.

2.4 Summary

We have thus seen how solid conductors can bend and buckle, and even shatter, if subjected to high current pulses. The discussion indicates that some kind of longitudinal stress is present.

In liquid conductors the results are not as clear cut as with solids, as pinch forces play an important part. However, we can see how the idea that a current tends to stretch itself (in the middle especially) could be used to explain many of those experiments qualitatively.

It is clear that in some of the described experiments the forces could be regarded as a result of transverse pinch forces, at least intuitively. However, from an engineering point of view the concept of longitudinal stretching could be very useful, since it directs the thinking along other lines than the Lorentz equation does. This view is supported by the fact that modern textbooks seldom, if ever, mentions experiments like those we have discussed above. The creative value of the conceptual images in engineering should not be underestimated.

When it comes to dense plasmas the forces are magnitudes greater than what could be produced by pinch forces. Though still not well understood, the phenomena of electrodynamic water explosions may have many applications, as we shall see in Chapter 5.

Chapter 3

Theoretical discussion

As we have seen, longitudinal forces can be a quite fruitful idea to understand some experiments. In this chapter we shall study the various theoretical approaches that have been made. A new investigation, based on Maxwell stresses, is given and is compared with the others.

Graneau has proposed the use of Ampère electrodynamics, based on Ampère's original formula, to account for the longitudinal stress. With the aid of a numerical method, finite current element analysis, he has estimated the stress predicted by Ampère's formula.

We begin with studying the way longitudinal stress actually arises from Ampère's formula and the problems with its computation. Maxwell stresses could also account for longitudinal stress. This is analyzed and compared with the Ampère electrodynamics. The relation between the approaches and the possibility of deriving Ampère's formula from classical electrodynamics is then discussed.

Relativistic electric fields and electric fields from surface charges have been advocated to predict longitudinal stress. This is analysed in the last sections, where the Ampère formula and other 2:nd order relativistic theories are put into context. The conclusions drawn from the discussion are summed up, ending the chapter.

3.1 Ampère electrodynamics

During the last debate, the main theory to account for longitudinal stress has been the Ampère electrodynamics, revived by Graneau. It is based on Ampère's original formula:

$$d\mathbf{F}_{Amp} = \frac{\mu_0 I_1 I_2}{4\pi r_{12}^2}[3(d\mathbf{l}_1 \cdot \hat{\mathbf{r}}_{12})(d\mathbf{l}_2 \cdot \hat{\mathbf{r}}_{12}) - 2(d\mathbf{l}_1 \cdot d\mathbf{l}_2)]\hat{\mathbf{r}}_{12}$$

Here $d\mathbf{F}$ is the force current element 1 ($d\mathbf{l}_1$) exerts on current element 2 ($d\mathbf{l}_2$); I_1 and I_2 are the electric currents in the circuits; μ_0 is the magnetic permeability; $d\mathbf{l}_1$

and $d\mathbf{l}_2$ are vectors in the direction of a current element, see Figure 3.1A; r_{12} is the distance between the current elements; and $\hat{\mathbf{r}}_{12}$ the unit vector from $d\mathbf{l_1}$ to $d\mathbf{l_2}$.

When deducing his formula, Ampère assumed that it would obey Newton's third law — i.e. that the forces between the current elements would be equal and opposite, and directed in the line between them. Ampère considered this a natural mathematical assumption, and it could neither be proved nor disproved from experiments with the interaction between closed circuits.

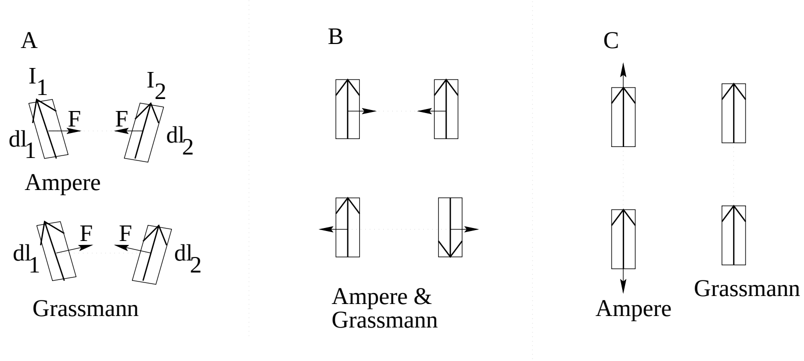

Figure 3.1: The forces between two current elements according to Ampère's and Grassmann's equations. (A) Forces between current elements. (B) Transverse forces between parallel elements. (C) Longitudinal forces between co-linear elements

As with the Grassmann law,

$$d\mathbf{F}_{Grass} = \frac{\mu_0 I_1 I_2}{4\pi r_{12}^2}[d\mathbf{l}_2 \times (d\mathbf{l}_1 \times \hat{\mathbf{r}}_{12})] = \frac{\mu_0 I_1 I_2}{4\pi r_{12}^2}[(d\mathbf{l}_2 \cdot \hat{\mathbf{r}}_{12})d\mathbf{l}_1 - (d\mathbf{l}_1 \cdot d\mathbf{l}_2)\hat{\mathbf{r}}_{12}]$$

two parallel current elements attract, and two anti-parallel repel, Figure 3.1B. But for co-linear elements the Ampère formula differs and predicts a repulsion, whereas the Grassmann force is zero, Figure 3.1C. (This was actually the criterion Grassmann used to derive his formula, as he thought the Ampère force behaved strangely.)

A long thin conductor would then experience a tension along itself, according to Ampère's formula. As we have seen, this was the argument used by Ampère to propose the hairpin experiment.

Actually, if the conductor is viewed as a bunch of filaments, current elements beside each other should attract, whereas they would repel their colinear neighbours, Figure 3.2A. The situation resembles very much stacking magnets side by side in a rectangular pipe, Figure 3.2B. The 'like' poles repel each other in the longitudinal direction (along the 'conductor'). The unlike poles in the transverse direction attract. As a result the 'conductor' experiences a longitudinal repulsion that is diluted, and a transverse pinch, just like a real conductor.

Imagining the magnets as circular, or circular flux, one understands why the analogy is good. (By folding each magnet into a circular flux loop, we get something that

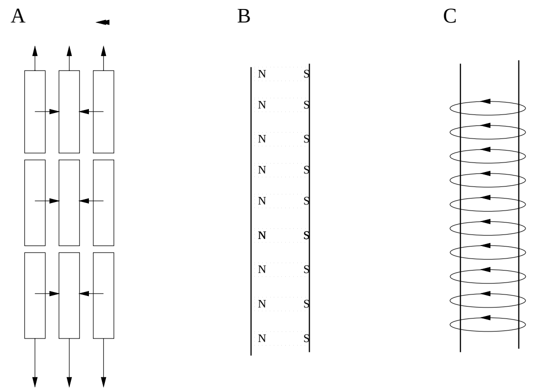

Figure 3.2: The 'stacked magnets' analogy. (A) Forces inside a conductor according to Ampère's force law. (B) Stacked magnets. (C) Magnetic flux around a current.

looks very much like the magnetic field associated with the current in a conductor, Figure 3.2C.)

Finite current element analysis

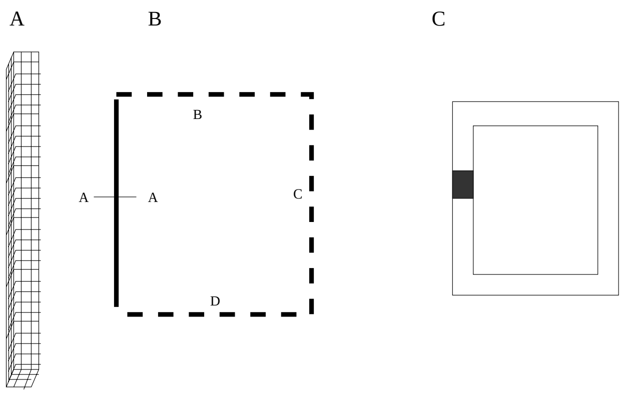

Figure 3.3: (A) Subdivision of a straight conductor. (B) Omitted parts. (C) Small segment of a circuit.

When trying to estimate the longitudinal stress, one runs into difficulties, as singularities arise when integrating the circuit around itself. This is due to the line current formulation. In order to avoid this Graneau has devised a method: 'Finite current element analysis'. The circuit is subdivided into filaments, Figure 3.3A, and integrated numerically. Subsequent iterations converge quite rapidly, as the repulsive force is 'diluted' by the side-by-side attraction.

Graneau has estimated the stress of a long straight conductor by calculating the forces all the current elements in the upper part exert on those in the lower part. This would give the stress across the middle section, A-A in Figure 3.3B.

The method is not unambiguous however. One question is if the stress is uniform or non-uniform, as the above approach mixes the two kinds of stresses.

Let us first consider non-uniform stress. This would arise if there were a difference in longitudinal force between two adjacent current elements; the difference would then be interpreted as stress, Figure 3.4A. One problem with the above method is the omission of parts of the circuit, the dotted lines in Figure 3.3B . The reason the other sides cannot be neglected is that a current element only experiences the sum of the forces from the rest of the circuit, as the element is not aware of from where the forces come.

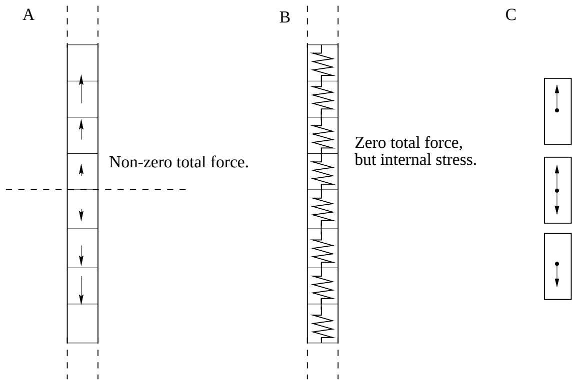

Figure 3.4: (A) Non-uniform stress. (B) Uniform stress. (C) Cancellation of forces.

How much is then neglected? The force on a short segment of a conductor, exerted by the rest of the circuit, was calculated. The longitudinal part of the force turned out to approach zero as the subdivision was made finer. Actually this is not as surprising as it seems. Only a non-uniform stress would reveal itself with this kind of analysis.

The stress is obviously more uniform in character. (The net force on an individual current element is zero in the axial direction, but the conductor is in a state of stress, as visualized by a series of strained springs in Figure 3.4B.) To predict such a stress the force between two adjacent current elements is essential. But these forces cancel out when forces from both sides of a current element are considered, as they are equal and opposite, Figure 3.4C. Thus the above reasoning does not give any clues into how to perform a calculation of the stress unambiguously. As we shall see though, there exist methods to estimate the stress.

3.2 Maxwell stresses and longitudinal magnetic pressure

During the debate in the 80s it has been tacitly assumed that longitudinal forces and stress cannot be predicted from classical electrodynamics. The arguments have been based on the fact that the Lorentz equation only predicts a transverse magnetic force. By focusing on the field properties rather than the charge carriers, I intend to show that Maxwell stresses, and thus classical electrodynamics, indeed can be interpreted to predict longitudinal stress.

The magnetic part of the Maxwell stress (the one of relevance in this discussion) is:

$$\mathbf{F} = -\frac{1}{2}\oint(\mathbf{B}\cdot\mathbf{H})d\mathbf{S} + \oint\mathbf{B}(\mathbf{H}\cdot d\mathbf{S})$$

Here $\mathbf{B}$ and $\mathbf{H}$ are the magnetic induction and field, respectively; and $d\mathbf{S}$ is the outward directed surface element. When integrating the expression around a body it yields the total force on it. For a moving charge we recover the Lorentz force. Forces on opposite surfaces that cancel under integration are also of interest as these predict stress in the body, this approach can e.g. be used to calculate the pinch pressure. As we shall see, the longitudinal forces reveal themselves in a similar way.

If we investigate the above expression we find that there exists a tension along the lines of flux, tending to shorten them, and a pressure between them, making them repel each other. Magnetic attraction and repulsion is easily visualized, Figure 3.5.

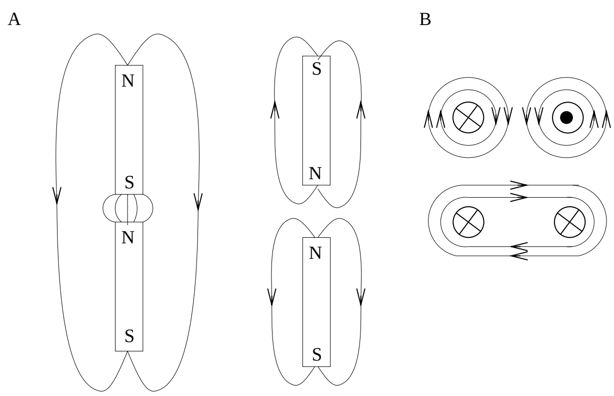

Figure 3.5: Attraction and repulsion between magnets as a result of Maxwell stresses: (A) Magnets. (B) Currents.

Solid conductors

Consider a straight conductor segment. In a straight conductor the magnitude of the forces only depends on the radial distance, whereas their direction depend on the orientation of the bounding surface. The Maxwell stress tends to split the conductor in the axial direction, the stress being strongest near the periphery, Figure 3.6A. In the tangential direction (along the flux lines) there is a tension. Thus two pieces of a axially split conductor attract each other, Figure 3.6B. In the radial direction a contraction exists which causes the pinch pressure, Figure 3.6C.

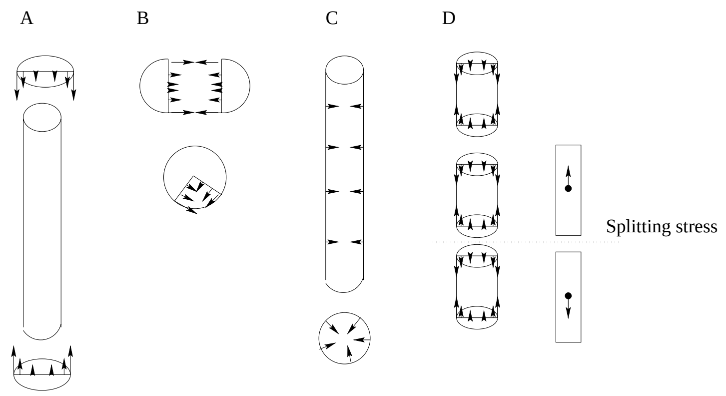

Figure 3.6: Maxwell stresses across different surfaces. (A) Axial stress, (B) Tangential stress, (C) Radial stress, (D) Splitting axial stress compared with repelling current elements.

From Figure 3.6D it is easy to see the similarity to the tension predicted by Ampère's equation. With a uniform current density (J) in the conductor we get:

$$\mathbf{H} = \frac{J\pi r^2}{2\pi r} = \frac{Jr}{2}, \quad r \leq R$$

for the magnetic field H at distance r from the center. R is the radius of the conductor. The magnitude of the stress σ is:

$$\sigma(r) = \frac{\mathbf{B}\cdot\mathbf{H}}{2} = \frac{\mu_0 J^2 r^2}{8} = \frac{\mu_0 I^2 r^2}{8\pi^2 R^4}$$

where we have introduced I as the total current.

Integrated across the axial surface we have,

$$\sigma_{mean} = \frac{\mu_0 I^2}{16\pi^2 R^2}$$

or

$$F = \frac{\mu_0 I^2}{16\pi}$$

for the mean splitting stress and the splitting force, respectively.

In this way one could imagine how this force could cause the initial separation in the multiarc generator, and how the combined effect of the radial shear and the axial tension could break the wire in Nasiłowski's experiment.

Liquid conductors

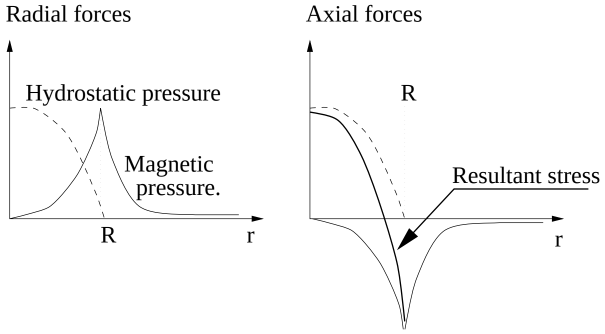

Figure 3.7: Radial and axial stresses in a liquid conductor.

Let us now examine a cylindric liquid conductor, as in Figure 3.6 (Regard the previous solid conductor as liquid). The radial magnetic pressure sets up stress in the conductor. This stress is balanced by the resulting hydrostatic pressure, when the system is at equilibrium. In the axial surface forces tending to compress the segment are set up. This compressive stress is greatest at the periphery and zero in the middle. The hydrostatic pressure, on the other hand, is zero at the periphery and has its maximum at the center. Thus these forces do not balance in the axial direction, Figure 3.7. (This transfer of radial pressure into axial is not present in solid conductors.) At the center the hydrostatic pressure dominates, causing the conductor to stretch itself there. At the periphery, where the magnetic pressure dominates, the conductor tends to contract.

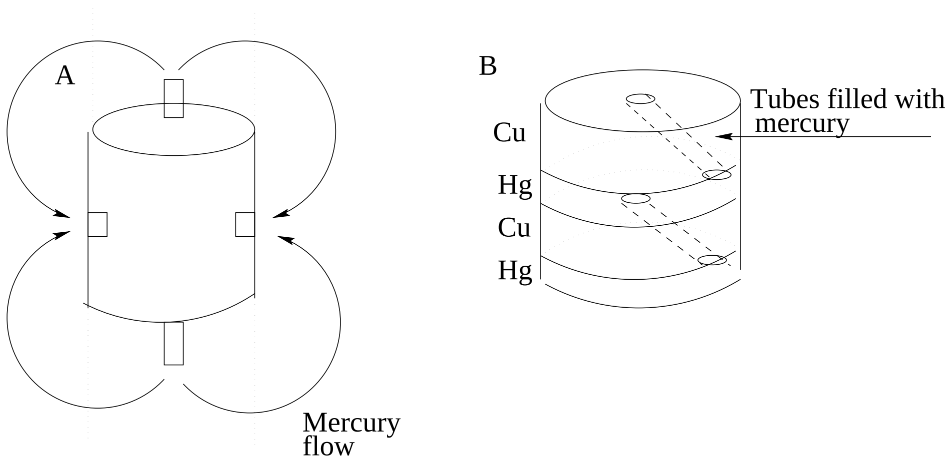

Figure 3.8: (A) Experiment to demonstrate the stretching in the middle. (B) Device to enhance the pressure.

This gives some insight into how the pinch pressure works. As the hydrostatical pressure is stronger than the axial Maxwell stress at the center, the conductor expands axially (if it has freedom to do so), at its center. Northrup devised an experiment to demonstrate the stretching in the middle, Figure 3.8A. The electric current flows in the axial direction. Due to the resulting pressures, mercury flows out from the center and recirculates back at the periphery. The pressure can be enhanced by adding it 'in series', as in Northrup's second experiment, Figure 3.8B. The pressure at the center of one mercury segment is connected to the periphery of the next one, et cetera. The pressure can be used to monitor the current with the aid of a manometer.

3.3 Relationship of the different force laws

Various attempts have been made to prove the equivalence or non-equivalence of the Lorenz-Grassmann force law and the Ampère force law. We are not going to digress into this discussion here. Most of the arguments have been about proving that no net longitudinal force exists on a part of a conductor. The main problem with these reasonings is that they cannot reveal a uniform stress — whether it exists or not in the law under consideration.

A more interesting question is the possibility of deriving Ampère's law from field theory. Rambaut [57] has derived Weber's electrodynamical potential from the relativistic potentials. From the Weber potential it is then straightforward to derive Ampère's formula. Assis [3] has shown that this is possible even with the modern idea of a current element (moving electrons and stationary ions) [3]. Other derivations of Ampère's law from relativistic potentials are also possible [56]. However, it is important to note that Neumann's force law (and potential) is a special case, applying only to interactions between closed circuits. It can be derived both from Ampère's and Grassmann's laws, and thus cannot be used to discriminate between the two, nor to prove their equivalence, although this is sometimes claimed.

The question arises whether it is necessary to use relativistic potentials to derive Ampère's force law. Could a relation be brought between Ampère's formula and the magnetic Maxwell stresses?

In order to understand the problem, let us consider the analogous case of a long charged rod. For a rod with radius R we have,

$$\mathbf{E} = \frac{\rho r}{2\epsilon_0}\hat{\mathbf{r}}, \quad r \leq R$$

where $\mathbf{E}$ is the electric field; ρ the volume charge density; ϵ_0 is the electric permittivity of vacuum; and $\hat{\mathbf{r}}$ is the unit vector in the radial direction. As above, r is the radial distance from the center.

An electrical Maxwell stress will act, tending to stretch the rod. With,

$$\mathbf{F} = -\frac{\epsilon_0}{2}\oint E^2 d\mathbf{S}$$

for the force on a circular cross-section, we have:

$$\mathbf{F} = -\frac{\rho^2 \hat{\mathbf{n}}}{8\epsilon_0} \int_0^{2\pi} \int_0^R r^2 r dr d\varphi = -\frac{R^4 \rho^2 \pi}{16\epsilon_0} \hat{\mathbf{n}}$$

Here $\hat{\mathbf{n}}$ is the unit normal to the circular surface, directed outwards from a volume element.

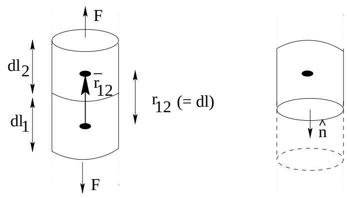

Figure 3.9: Adjacent elements in the rod.

If we now regard this separating force as being the repulsion between two adjacent volume elements, we should arrive with something that looks like the Coulomb law, with some geometrical correction factor. Hence:

$$\mathbf{F} = -\frac{\pi R^2 \rho dl}{dl} \cdot \frac{\pi R^2 \rho dl}{dl} \cdot \frac{\hat{\mathbf{n}}}{16\pi\epsilon_0} = -\frac{Q_1}{dl} \cdot \frac{Q_2}{dl} \cdot \frac{\hat{\mathbf{n}}}{16\pi\epsilon_0} = -\frac{Q_1 Q_2}{4\pi\epsilon_0 r_{12}^2} \hat{\mathbf{n}} \cdot \frac{1}{4}$$

(Note that r_{12} means the distance between two current elements, and should not be confused with r.) Thus the view of Maxwell stress across the surface, and the view of line-charges repelling each other show a close relationship.

Now substitute the rod by a long conductor. The magnetic field from the current is:

$$\mathbf{H} = \frac{Ir}{2\pi R^2} \hat{\phi}$$

Here I is the electric current and $\hat{\phi}$ is the unit vector in the tangential direction. The Maxwell stress on the circular surface is:

$$\mathbf{F} = -\frac{\mu_0}{2} \oint H^2 d\mathbf{S} = -\frac{\mu_0 I^2 \hat{\mathbf{n}}}{8\pi^2 R^4} \int_0^{2\pi} \int_0^R r^2 r dr d\varphi =$$

$$= -\frac{\mu_0 I^2}{16\pi} \hat{\mathbf{n}} = -\frac{Idl}{dl} \cdot \frac{Idl}{dl} \cdot \frac{\mu_0}{16\pi} \hat{\mathbf{n}} = -\frac{\mu_0 I^2}{4\pi} \cdot \frac{dl_1 dl_2}{r_{12}^2} \hat{\mathbf{n}} \cdot \frac{1}{4}$$

where the left part of the expression is recognized as the Ampère force between two colinear current elements. As mentioned before, Ampère had to assume that the force was in the line in between the current elements (and thus obeying Newton's third law), as he couldn't derive it from experiment. Let us now assume that the force between adjacent colinear current elements actually is

$$-\frac{\mu_0 I^2}{4\pi} \cdot \frac{dl_1 dl_2}{r^2} \hat{\mathbf{n}}$$

This is quite reasonable as we in the electrostatical case have the Coulomb force in the corresponding expression. Could this be used to uniquely define a force law, e.g. Ampère's? As we shall see, the answer is quite in the affirmative.

Analysing Ampère's experiments, without the assumption of Newton's third law, Whittaker [67] derived the following expression for the most general force law:

$$d\mathbf{F} = \frac{\mu_0 I_1 I_2}{4\pi r_{12}^2}[3(B+1)(d\mathbf{l}_1\cdot\hat{\mathbf{r}}_{12})(d\mathbf{l}_2\cdot\hat{\mathbf{r}}_{12})-(2+B)(d\mathbf{l}_1\cdot d\mathbf{l}_2)]\hat{\mathbf{r}}_{12}+A(d\mathbf{l}_1\cdot\hat{\mathbf{r}}_{12})d\mathbf{l}_2-B(d\mathbf{l}_2\cdot\hat{\mathbf{r}}_{12})d\mathbf{l}_1$$

A and B are here numerical constants that cannot be evaluated from Ampère's own experiments. However, from the reasoning about the Maxwell stress above we arrived with

$$-\frac{\mu_0 I^2}{4\pi}\cdot\frac{dl_1 dl_2}{r_{12}^2}\hat{\mathbf{n}} = \frac{\mu_0 I^2}{4\pi}\cdot\frac{dl_1 dl_2}{r_{12}^2}\hat{\mathbf{r}}_{12}$$

for the force between adjacent co-linear current elements. This is equivalent to having

$$3(B+1)-(2+B)+A-B=1$$

or

$$A=-B$$

We thus have a formula that is symmetric in $d\mathbf{l}_1$ and $d\mathbf{l}_2$. Whittaker arrived with the same result from considerations of linear force balance (from aestethic rather than experimental considerations).

Is there any way we could determine the remaining constant? Aspden [1] noted another condition that can be put on the force law, by analysing an experiment performed by Trouton and Noble [66]. (They found that a capacitor showed no tendency to turn when in linear motion transverse to its suspension.) Aspden concludes:

> 'There is no interaction torque out of balance between anti-parallel current elements . . . To satisfy [this] observation, terms other than those in $\hat{\mathbf{r}}_{12}$ must cancel when $d\mathbf{l}_1$ is equal to $-d\mathbf{l}_2$.'

This means that $A=B$ and, as we already have $A=-B$, we get $A=B=0$, or Ampère's force law:

$$d\mathbf{F}_{Amp} = \frac{\mu_0 I_1 I_2}{4\pi r_{12}^2}[3(d\mathbf{l}_1\cdot\hat{\mathbf{r}}_{12})(d\mathbf{l}_2\cdot\hat{\mathbf{r}}_{12})-2(d\mathbf{l}_1\cdot d\mathbf{l}_2)]\hat{\mathbf{r}}_{12}$$

(Strictly speaking, one may question whether the current elements in metallic conduction currents, and those in a capacitor moving sideways can be considered equivalent, as Aspden's reasoning implies. It is a remarkable fact though, that when so is done, one arrives with Ampère's formula. Anyway, the value of the second constant doesn't affect the repulsion between co-linear current elements.)

Thus we more or less experimentally (as the Maxwell stress can explain the longitudinal forces) have derived Ampère's formula, without the additional assumption of the law obeying Newton's third law. Of course, taking the route over field theory doesn't make the derivation as clear cut as Ampère's. In a sense we have shown that Ampère's formula can be inferred from field theory[1]. The above reasoning is not intended to be any rigorous proof, but rather to show the intuitive value of both the views.

But from the reasoning it is clear that Graneau's finite current element analysis overestimates the tension, even if just the interaction between two filamentary current elements is considered. (The stress predicted should be the same as the mean Maxwell stress, but Graneau reports a stress about ten times greater, from his calculations.) The same problems arise when trying to calculate electric stresses with Coulomb's law.

We can see how Ampère's law, when it comes to forces in a conductor, plays the same part as Coulomb's law does in electrostatics. Maxwell stresses and Ampère's formula turn out to be two complementary views — one focuses on the field properties and the other on current elements — Maxwell stresses being the simplest to use in my view.

'Action at a distance' theories

In what respects does Ampère electrodynamics then differ from field theory? When it comes to longitudinal forces they are obviously saying quite the same thing. But is there anything else in the Ampère electrodynamics that is interesting and not apparent in classical electrodynamics?

From Ampère's law several action at a distance theories have been derived [45]. The most general of these is the one derived by Moon and Spencer [46]. It is a 2:nd order theory, i.e. it includes 2:nd order relativistic effects, such as the relativistic electric field. Several paradoxes in electromagnetic induction (dealing with e.g. homopolar and moving boundary induction) are easily solved within this theory [48, 49]. Thus being on the threshold between classical and relativistic electrodynamics, it may be useful for calculations.

Another 2:nd order theory is the Darwin formulation, which is easily derived from the relativistic potentials [39]. It has shown to be useful in plasma physics calculations [41], though it does not, as the Moon and Spencer formulation, include radiation effects.

[1] As it turns out, we have been using relativistic arguments without invoking relativity theory, as Trouton-Noble's experiment is a consequence of special relativity

An overview of the different theories is given below:

Potentials:

$$P_{Darwin} = \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}}\{-1 + \frac{1}{2c^2}[\mathbf{v}_1 \cdot \mathbf{v}_2 + (\mathbf{v}_1 \cdot \hat{\mathbf{r}}_{12})(\mathbf{v}_2 \cdot \hat{\mathbf{r}}_{12})]\}$$

$$P_{Weber} = \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}}\{-1 + \frac{\dot{r}_{12}}{2c^2}\} = \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}}\{-1 + \frac{1}{2c^2}(\mathbf{v} \cdot \hat{\mathbf{r}}_{12})\}$$

Force laws (charge formulation):

$$\mathbf{F}_{Darwin} = \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}^2}\{\hat{\mathbf{r}}_{12} - \frac{1}{2c^2}[\mathbf{v}_1 \cdot \mathbf{v}_2 + 3(\mathbf{v}_1 \cdot \hat{\mathbf{r}}_{12})(\mathbf{v}_2 \cdot \hat{\mathbf{r}}_{12})]\hat{\mathbf{r}}_{12} + \frac{1}{2c^2}[(\mathbf{v}_1 \cdot \hat{\mathbf{r}}_{12})\mathbf{v}_2 + (\mathbf{v}_2 \cdot \hat{\mathbf{r}}_{12})\mathbf{v}_1]\}$$

$$\mathbf{F}_{Weber} = \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}^2}\{1 - \frac{\dot{r}_{12}^2}{2c^2} + \frac{r_{12} \cdot \ddot{r}_{12}}{c^2}\}\hat{\mathbf{r}}_{12} = $$

$$= \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}^2}\{1 + \frac{1}{c^2}[\frac{-3}{2}(\mathbf{v} \cdot \hat{\mathbf{r}}_{12})^2 + (\mathbf{v} \cdot \mathbf{v}) + \mathbf{r}_{12} \cdot (\mathbf{a}_1 - \mathbf{a}_2)]\}\hat{\mathbf{r}}_{12}$$

$$\mathbf{F}_{Gauss} = \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}^2}\{1 + \frac{1}{c^2}[\frac{-3}{2}(\mathbf{v} \cdot \hat{\mathbf{r}}_{12})^2 + (\mathbf{v} \cdot \mathbf{v})]\}\hat{\mathbf{r}}_{12}$$

$$\mathbf{F}_{MoSp} = \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}^2}\{\frac{1}{c^2}[\frac{-3}{2}(\mathbf{v} \cdot \hat{\mathbf{r}}_{12})^2 + (\mathbf{v} \cdot \mathbf{v})]\hat{\mathbf{r}}_{12} - r_{12}\frac{\partial}{\partial t}\mathbf{v}(t - \frac{r_{12}}{c})\} - \frac{q_2}{4\pi\epsilon_0}\frac{\partial}{\partial r_{12}}[\frac{1}{r_{12}}q_1(t - \frac{r_{12}}{c})]\hat{\mathbf{r}}_{12}$$

Force laws (current element formulation):

$$d\mathbf{F}_{Amp} = \frac{\mu_0 i_1 i_2}{4\pi r_{12}^2}[3(d\mathbf{l}_1 \cdot \hat{\mathbf{r}}_{12})(d\mathbf{l}_2 \cdot \hat{\mathbf{r}}_{12}) - 2(d\mathbf{l}_1 \cdot d\mathbf{l}_2)]\hat{\mathbf{r}}_{12}$$

$$d\mathbf{F}_{Grass} = \frac{\mu_0 i_1 i_2}{4\pi r_{12}^2}[d\mathbf{l}_2 \times (d\mathbf{l}_1 \times \hat{\mathbf{r}}_{12})] = \frac{\mu_0 I_1 I_2}{4\pi r_{12}^2}[(d\mathbf{l}_2 \cdot \hat{\mathbf{r}}_{12})d\mathbf{l}_1 - (d\mathbf{l}_1 \cdot d\mathbf{l}_2)\hat{\mathbf{r}}_{12}]$$

$$\mathbf{F}_{Neu} = -\frac{\mu_0 I_1 I_2}{4\pi}\oint\oint\frac{d\mathbf{l}_1 \cdot d\mathbf{l}_2}{r_{12}^2}\hat{\mathbf{r}}_{12}$$

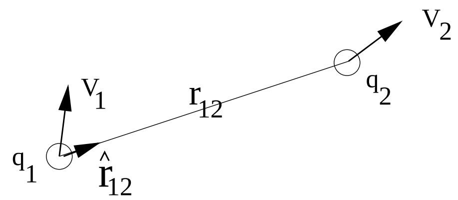

Figure 3.10: The meaning of the different vectors.

Here $\mathbf{F}$ is the force charge 1 (q_1) exerts on charge 2 (q_2); $\mathbf{v}_1$ and $\mathbf{v}_2$ are the velocities of the charges; $\mathbf{v} = \mathbf{v}_1 - \mathbf{v}_2$ their relative velocity; $\mathbf{a}_1$ and $\mathbf{a}_2$ are their accelerations; $\mathbf{r}_{12}$ is the vector from charge 1 (q_1) to charge 2 (q_2); $\dot{\mathbf{r}}_{12} = (\mathbf{v} \cdot \hat{\mathbf{r}}_{12})\hat{\mathbf{r}}_{12}$ is its first derivative, or the relative velocity projected on the line between the charges; and $\ddot{\mathbf{r}}_{12}$ the second derivative, which can be expanded into and expression of the velocities and accelerations.

As can be seen, Gauss' and Weber's laws differ only in the acceleration terms. Consequently they are equivalent in magnetostatics. As Gauss formula doesn't contain any acceleration terms, it alone cannot predict electromagnetic induction, whereas Weber's law can.

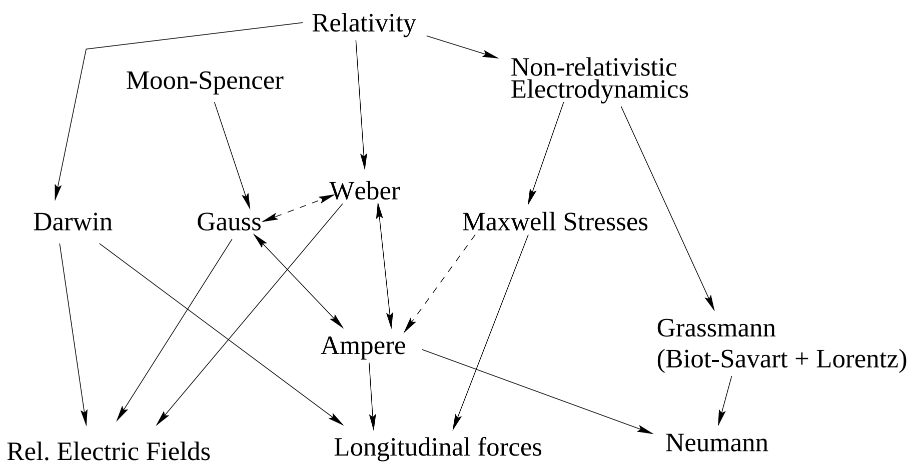

Figure 3.11: Relationships of the force laws.

3.4 Relativistic electric fields

Ivezić suggested that relativistic electric fields could account for longitudinal forces. What then, is the relativistic electric field? In classical electrodynamics it is assumed that a conductor with an equal number of positive a negative charges (neutral) is experienced as charge neutral, even though the electrons are moving. Relativistically it cannot be so.

Consider a long straight conductor that is charge neutral, and then switch on a current. We still have the same number of electrons. But as they are moving, the spacing between them shrink (the Lorentz contraction) and we observe a charge density from them that is higher than when at rest. Let the charge density for positive and negative charge at rest be: $\rho^+ = \rho_0$ and $\rho^- = -\rho_0$ respectively. Then, when the electrons are moving we observe, in the laboratory frame, a charge density from them that is $\rho^- = -\gamma\rho_0$. For the total observed charge we have:

$$\rho = \rho_0 - \gamma\rho_0 = (1-\gamma)\rho_0 \approx -\frac{v^2}{2c^2}\rho_0$$

where v is the drift velocity of the electrons, and

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

Hence the conductor behave as if it had an excess negative charge, and is surrounded by an 'electric' field, the relativistic (or 2:nd order) electric field. Normally this field is much weaker than the magnetic field. Between two metallic 'neutral' conductors the resultant force will be proportional to $\frac{v^4}{c^4}$, as the field is produced by the $\frac{v^2}{c^2}$ fraction in one of the conductors, and experienced only by a similar fraction in the other conductor.

However, when it comes to forces on the charges within one and the same conductor, the situation is quite different. Then the relativistic electric field is of the same order as the self-induced Hall effect (which is caused by the electrons cutting the flux of their neighbours). Thus when it comes to charge distributions within plasmas, the relativistic electric field could be important.

But could these fields cause longitudinal stress in a metallic conductor, as has been suggested? Within the conductor we have:

$$E_{rel} = -\frac{\rho_0 r}{2\epsilon_0}\frac{v^2}{2c^2}\hat{\mathbf{r}}$$

where ϵ_0 is the electric permittivity of vacuum, and r the radial distance from the center of the conductor. The maximum stress is at the periphery, where $r = R$. Observing that $\rho_0 v \cdot \pi R^2 = I$ and $\mu_0 = \frac{1}{\epsilon_0 c^2}$ we get:

$$E_{rel} = -\frac{\mu_0 I v}{4\pi R}\hat{\mathbf{r}}$$

for the field at the periphery. The electric Maxwell stress is

$$\sigma = \frac{\mathbf{E}\cdot\mathbf{D}}{2} = \frac{\epsilon_0 E^2}{2} = \frac{\epsilon_0 I^2 \mu_0^2 v^2}{32\pi^2 R^2} = \frac{\mu_0 I^2}{8\pi^2 R^2}\cdot\frac{\epsilon_0\mu_0 v^2}{4} = \frac{\mu_0 I^2}{8\pi^2 R^2}\cdot\frac{v^2}{4c^2}$$

where the left part of the expression on the righthand side is the same as the magnetic Maxwell stress we have studied earlier. As the drift velocity is very small, the Maxwell stress from the relativistic electric fields is negligible compared to the magnetic Maxwell stress.

3.5 Electric fields from surface charges

It has been suggested that electric fields from surface charges may be of importance in some of the experiments with forces between parts of the same conductor [38]. The electric fields from surface charges in an electric circuit are essential for answering questions like:

- 'How does a conduction electron know how to turn at a corner in a wire?'
- 'Does the electric field caused by e.g. a battery exert a force on a charge outside the circuit, or on the circuit itself?'
- 'How is energy fed into a steady current in a conductor?'

Yet electric fields from surface charges are seldom mentioned in textbooks, causing a jump in the narrative between electrostatics and magnetostatics[2].

[2] An exception is Jefimenko's excellent book [40].

Let us consider a simple example, to see whether we have to count with these fields or not — i.e. if they are strong enough to cause a substantial Maxwell stress, or if they can cause interaction forces between different parts of a circuit of a magnitude comparable to the magnetic forces. A circular circuit with a point dipole battery, see Figure 3.12, has been examined by Heald [32].

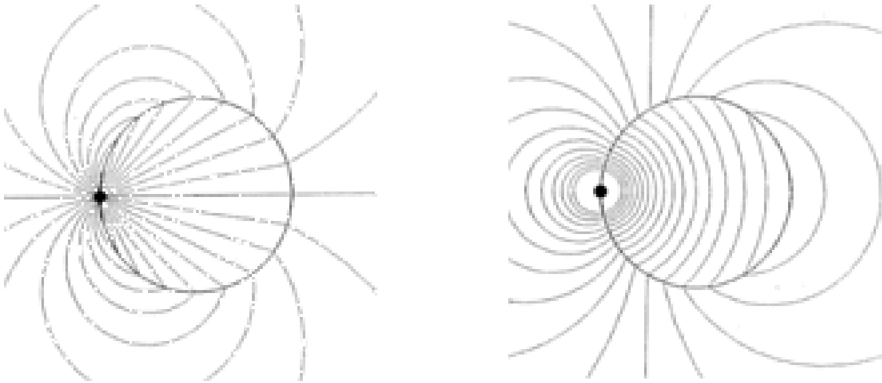

Figure 3.12: (A) Equi-potential lines (and also lines of energy flux), (B) Electric field in the round circuit.

The battery is represented by the point, and the circuit by the darker circle. The magnitude of the electric field is:

$$E = \frac{V_0}{\pi r_0}$$

where V_0 is the potential drop across the battery, and r_0 the distance from origin (where the battery is). To sustain this field, and thus the conduction, charges are set up on the surfaces of the wire. The radial component of this field can exceed the axial field (which drives the conduction) substantially, depending on the geometry of the circuit. Now, could this field cause a longitudinal Maxwell stress? The electric Maxwell stress is given by:

$$\sigma = \frac{\epsilon_0 E^2}{2} = \frac{\epsilon_0 V_0^2}{2\pi^2 r_0^2}$$

Could this stress cause the breaks in Nasiłowski's experiment? Typically for Nasiłowski's experiment we have $V_0 = 30kV$, $r_0 \approx 1m$ (as the thin wire broke far from the battery). This gives,

$$\sigma \approx 4 \cdot 10^{-4} N/m^2$$

which is negligible. Only very close to the battery, or where we have sharp bends, we could expect a higher stress.

Then what about forces between different parts of a circuit? According to Jefimenko, electric fields from surface charges can influence precision measurements of such forces [40]. A condition is that the capacity between different parts of a circuit is high compared to the inductance. In most of the experiments we have studied this has not been the case. Presence of materials with high electric permittivity (such as water) could enhance the forces substantially. Possibly, forces from surface charges could be of interest in the water-arc explosion experiments.

3.6 Summary

In this chapter we have analysed the Ampère electrodynamics and compared it with the Maxwell stress approach. We have seen how these are two sides of the same coin — one focusing on charge carriers, and the other on the field properties. The forces predicted are of the same magnitude as the well known pinch forces, but act in other directions. As Graneau's 'Finite current element analysis' mixes uniform and non-uniform stress, it over-estimates the stress, explaining why the longitudinal stress has been calculated to about ten times the stress from pinch forces.

The above approach could explain the phenomena observed in solid and liquid conductors. When it comes to dense plasmas, the forces are much stronger, and probably of different origin.

Relativistic electric fields may be important when it comes to the charge distribution in plasmas, but are far too weak to cause any longitudinal stress, nor any measurable forces between different parts of a metallic conductor.

Electric fields from surface charges may cause detectable forces in circuits with a large capacity to inductance ratio. However, that is not the case in the solid and liquid conductor experiments studied here.

Chapter 4

Analysis of experiments

In this chapter we will apply the Maxwell stress approach to some of the experiments, and compare the theoretical and experimental data.

4.1 The multi-arc generator

Let us begin with analysing the multi-arc generator. Segments of a copper rod are stacked vertically. A spring with a light pressure keeps them in contact. When a current pulse is passed the pieces separate.

A longitudinal repulsion acts between neighbouring segments, causing them to repel each other. This repulsion is balanced by gravity and the light spring pressure. The downward force is of course greatest on the bottom piece, as it has to carry the gravity of all the other pieces. In order to separate the bottom piece from its upper neighbour all the segments above it have to be levitated by its repulsion. The situation is similar to magnets stacked side by side in a box, Figure 4.1.

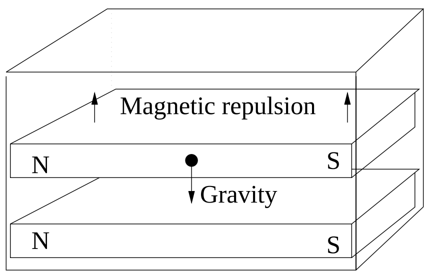

Figure 4.1: Magnets stacked in a box. The upper magnet levitates due to the magnetic repulsion.

Is the Maxwell stress strong enough to separate the pieces? Let us consider the bottom piece and calculate the required forces:

Rod type	Rod volume	Rod mass	Rod weight	Typical current
50 2mm Al	$3.2cm^2$	8.5g	0,08N	6kA
50 2mm Cu	$3.2cm^2$	29g	0,3N	6kA
50 2cm Al	$32cm^2$	86g	0,8N	25kA
50 2cm Cu	$32cm^2$	290g	3N	25kA

The forces that the Maxwell stress has to overcome are in the range 0.08-3.0N, plus the additional light spring pressure. In the cases where aluminium and copper segments were mixed, the force should be somewhere in between. The force from the Maxwell stress is:

$$F_{Maxwell} = \frac{\mu_0 I^2}{16\pi} = \frac{10^{-7}}{4} I^2$$

With $i = 6 - 25kA$ we have $F_{Maxwell} = 0.9 - 15.6$N, well enough to separate all the segments. The excess part of the force compresses the spring until the forces balance. The above reasoning suggests that if a weak current is used, only segments higher up will separate.

4.2 Nasiłowski's experiment

Let us now study Nasiłowski's experiment. Could the Maxwell stress be responsible for the wire breaks? As we have seen in the theoretical discussion, the wire should be subjected to a radial pinch pressure and an axial stress. In the MIT experiments Graneau used 1.2mm Al wires, and currents in the range 5-7kA. The wires shattered at about 6kA. The maximum of the longitudinal Maxwell stress would be:

$$\sigma(r) = \frac{\mu_0 I^2 r^2}{8\pi^2 R^4} = 1.6N/mm^2$$

Here r is the radial distance from the center or the conductor, and R the radius of the conductor. In addition to this a pinch pressure of the same magnitude acts. As the stress profile in the wire is quite complicated, we could expect the maximum stress across any surface to be of same the order as the sum, i.e $\approx 3-4N/mm^2$. This is not a great stress. However, the wire was severely weakened by being heated to almost its melting point before it broke. Aluminium at room temperature has a tensile strength of about $70N/mm^2$. At 600 $^\circ$C it is reduced to roughly $25N/mm^2$ [2]. Of course there is still a gap to $4N/mm^2$. It may be that the temperature was substantially higher in some part of the conductor. The shock of the stress, as it was applied quite rapidly, in about 1ms, may have contributed to shatter the wires. Nasiłowski used weaker currents than Graneau. His wires also showed more signs of melting at the surface, a fact that supports the above arguments. (Nasiłowski's copper wires were sometimes heated to 900-1000 $^\circ$C — almost to the melting point.)

In order to make more clear conclusions, more accurate data on wire temperatures and tensile strength is needed.

4.3 Railgun recoil

Let us now examine the railgun deformation. The rails deformed, as if subjected to compressive forces, Figure 4.2A. The situation is more clear cut in the impulse

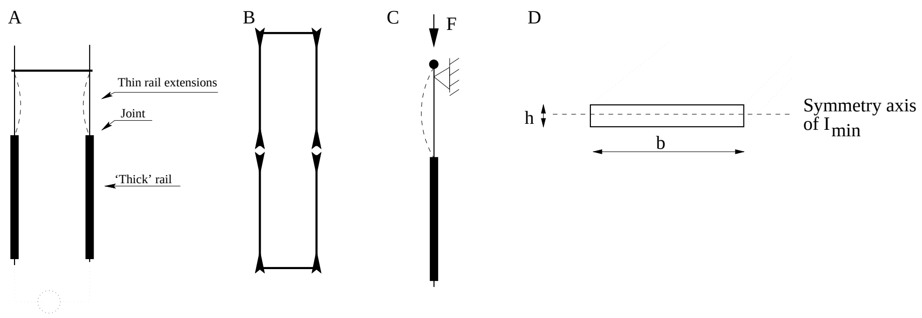

Figure 4.2: (A) Compressive forces in the railgun. (B) Separation forces at the ends in the impulse pendulum. (C) Euler breaking. (D) Section of the rail extensions.

pendulum experiment, as no influences from the corners can be expected in that case, Figure 4.2B. However, from the Maxwell stress approach, we expect the compressive forces in the two cases to behave similarly.

How great a force would be needed to deform the thin rails in Graneau's experiment? Obviously we have a situation with Euler breaking. As one end is free to move, but supported from one side, we have Euler's second case, Figure 4.2C. The force is given by:

$$F_{Break} = \frac{\pi^2 E I_{min}}{(\gamma L)^2}$$

The notation is quite confusing. E here means the Young modulus; I_{min} the least area momentum of inertia; L is the length of the rod (or rail in this case); and γ is a geometric factor depending on how the rod is fixed. The least area moment of inertia, see Figure 4.2D, is:

$$I_{min} = \frac{bh^3}{12}$$

The width (b) is 12.7mm and the thickness (h) of the rail extensions is said to be much smaller than that of the thicker 1.3mm (0.5") rails [21]. Let us assume that h was at least 0.5mm. With $I_{min} = 0.14mm^4$, $L = 300mm$, $\gamma = 1$ and $E = 70$ and $210kN/mm^2$ for aluminium and iron, respectively, we have:

$$F_{break} = 1.1N \text{ (Aluminium) and } F_{break} = 3.2N \text{ (Iron).}$$

How does this compare with the Maxwell stress? The force would be:

$$F_{Maxwell} = \frac{\mu_0 I^2}{16\pi}$$

where I now means the current in the circuit and μ_0 is the magnetic permeability of vacuum. Graneau reports he used currents in the range 10-100kA [21], which gives

$$F_{Maxwell} = 2.5 - 250N$$

well enough to cause severe deformation of the rails.

The impulse pendulum

Perhaps we should also consider the impulse pendulum situation to elucidate how longitudinal forces and pinch forces act. As the ends are in front of each other, without corners in the neighbourhood, the situation is more straightforward to visualize, see Figure 4.3.

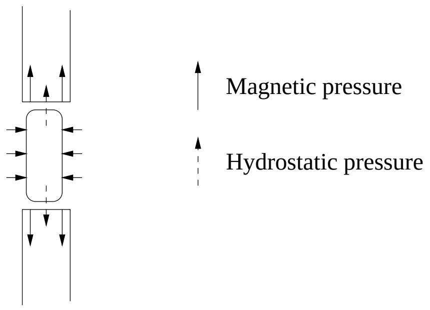

Figure 4.3: The forces at the interface between the rods.

At the end of the rods a longitudinal Maxwell stress acts. This tends to separate the rods, the force being strongest at the periphery, and zero at the center.

In addition to this a pinch pressure acts hydrodynamically. As the rods separate a small distance, an arc strikes between the ends. The pinch pressure in this arc causes it to contract radially, and thus stretch in the middle, transferring radial pressure into axial. This hydrodynamical pressure also acts on the ends of the rods, mechanically. This pressure is of course greatest at the center of the rod.

Thus the force on each end is a sum of the longitudinal Maxwell stress, and the hydrodynamical arc pressure. If we regard the two rods in contact as one single rod, it is clear that it should 'break' at the interface, as its tensile strength is weakest there.

4.4 Movement of conductors

An interesting question is if the longitudinal Maxwell stress is perfectly balanced all along a conductor, or if it could contribute to the forces between two parts of a circuit.

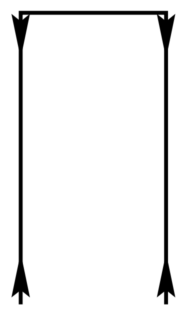

Figure 4.4: End force balanced by force in the corner.

Let us consider the impulse pendulum, Figure 4.4. Can we, for example, be sure the the Maxwell stress from the end faces of a pi-frame is perfectly balanced by the Maxwell stress in the corners, where the conductor is no longer straight? Or to put it more clear cut: When it comes to forces between two parts of the same circuit, does the Grassmann and the Ampère force laws differ quantitatively? In the impulse pendulum case, which can be regarded as generic, very precise calculations have been performed by Moyssides [50]. The predicted forces seem to be equal, the small differences being due to the numerical integration.

Where we have an asymmetry, a change in diameter, or a bend, we necessarily have a current that moves transversely with regard to the current in its neighbourhood, and consequently Lorentz forces. It is thus the asymmetry in the Lorentz forces that cause the movements, as long as the circuit doesn't deform itself axially. Typical asymmetry movements are Hering's crooked wire experiment and the copper submarine. Typical axial deformations are railgun bucking, the Multiarc generator (at least far from the ends), and Nasiłowski's wire fragmentation.

4.5 Flux-stretching

Many of the asymmetry movements can be visualized as stretching of the magnetic flux (which in those cases is equivalent to considering the Lorentz forces). This was Hering's approach. If the current is uniform all the repulsions balance. Movements manifest whenever the flux density is weaker at some part of the conductor.

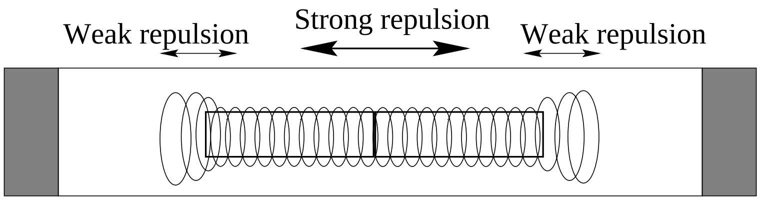

Figure 4.5: Nonuniform magnetic flux causing the separation of the bars.

Consider e.g. the separating bars,Figure 4.5. As the current diverges outside the faces of the rods, the flux density is weaker there. The more concentrated flux at the middle overpowers the weak flux at the ends, causing the two bars to separate. Similar arguments can be applied to the chain, the copper submarine, and Hering's wire movement.

Chapter 5

Applications

Having thus discussed the different experiments and theories we will now focus on some possible applications.

5.1 High-current limiting and switching

The application of the multi-arc generator in current limiting is obvious. For low currents it is a good conductor. As the current rises into the kA-range arcs develop, first in some places, and as the current rises, between all the sections. The resistance thus increases with the current.

As suggested by Ruscak and Bruce, this could make it useful as a self-resetting current limiter in the kA-range. In lightning arresters, the multiarc generator could be substituted for the resistance blocks, used to dissipate the lightning energy [62].

5.2 Liquid metal pumping

The following liquid metal pump was invented by Hering and his coworkers, to create a unidirectional flow in a furnace, Figure 5.1A. A strong circulation of metal was observed when current (DC or AC) was switched on. The direction of the electric current did not influence the metal flow [33, 34]. As the Ampère formula predicts the force to be in the line in between the current elements, one component of the force must be in the longitudinal direction (in the direction of current flow), as the currents are inclined towards each other. As the net longitudinal force from the middle hollow conductors isn't balanced by forces at the outer ones, Figure 5.1B, the circulation of metal results. If the construction is slightly modified it can pump metal between different furnaces. This may e.g. be achieved by inserting a conducting wall at the dotted line in Figure 5.1A.

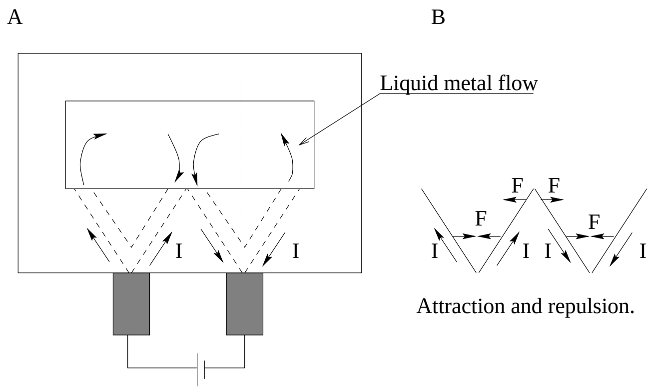

Figure 5.1: (A) Hering's liquid metal pump. (B) Seat of forces.

5.3 Punching and metal engineering

Water-arc explosions have long been used in metal engineering to create high pressures in 'electrohydraulic forming'. An arc is discharged in water, creating a high pressure which then shapes a metal sheet. The development has been based on trial and error, as the nature of the explosions has not been well understood. The water

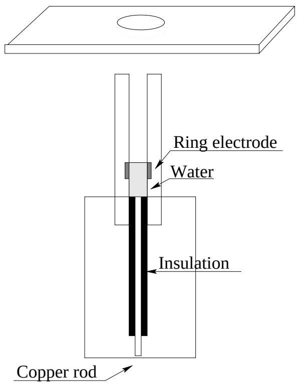

Figure 5.2: Punching of metal with water-arc explosions.

explosions can also be used for punching, Figure 5.2. In one of Graneau's water-arc experiments a 3.8g water column punched a clean 13mm hole in a 6.4mm thick aluminium plate, after having travelled 10cm in the air [25]. The impact velocity was around 1000m/s.

5.4 The electrodynamic explosion motor

The electrodynamic explosions could be used to drive an explosion motor, Figure 5.3. An arc discharge causes a directed explosion in the water. The impact of the water

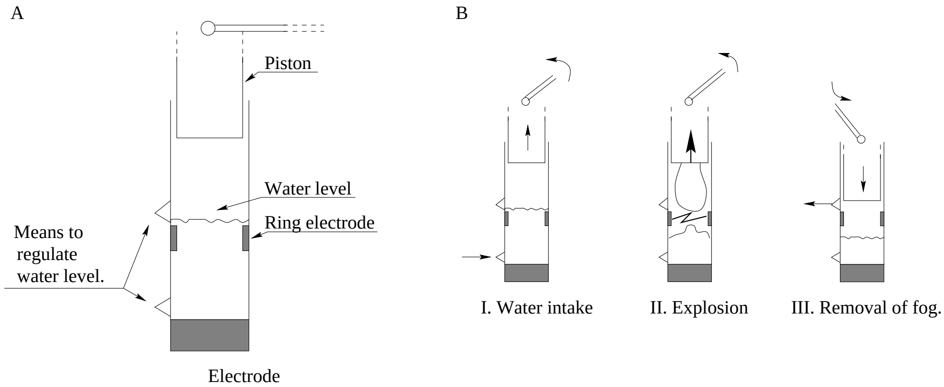

Figure 5.3: The electrodynamic explosion motor.

onto the piston creates a torque on a crankshaft. The piston need not even fit in tight with the cylinder, as it is the impact of the water, and not gas pressure, that causes the force. This motor would have several advantages:

- The force on the piston can act when it exerts the maximum torque on the crankshaft.
- The force is directed, as it is the impact of the expelled water and not thermal pressure that produces the forces.
- High pressures, 20'000-40'000 atm, can be created even in a small motor. This could give a high power/weight ratio.

One disadvantage would be:

- Some energy is lost as joule heating in the leads, and due to some electrolytic conduction. Clearly, energy is also lost due to the ionization.

5.5 Water-arc jet propulsion

Based on his own experiments with water explosions and the liquid mercury fountain, Graneau proposed the following device for water-arc jet propulsion [23], Figure 5.4. Water is expelled from the rod by the electrodynamic repulsion (explosions), and is continuously supplied from behind the rod. The device has the advantage over Magnetohydrodynamic (MHD) propulsion that it does not need any superconducting magnets. MHD propulsion is based on the Lorentz forces acting on an electrolytic current in a strong magnetic field, the immense magnets needed being its main drawback.

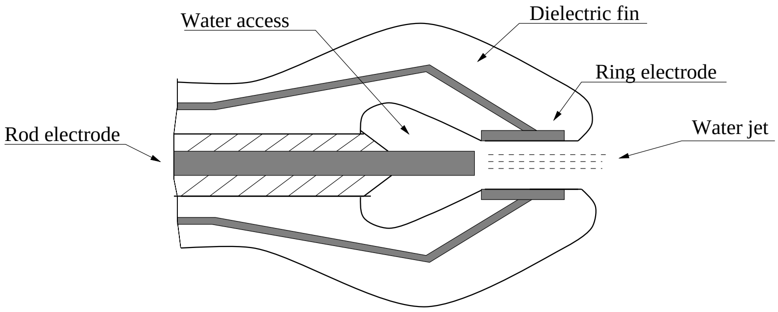

Figure 5.4: Water-arc jet for the propulsion of ships and submarines.

What about the efficiency of the jet? The thrust force is given by:

$$F = \frac{\mu_0}{4\pi} k I^2$$

where μ_0 is the magnetic permeability of vacuum, I the current and k a performance index. For the water-arc k is about 1000 or more. As a comparison the railgun has $k \approx 7$. Currents of the order of kA can produce a useful thrust. According to Graneau, water heating will not be a serious problem, whereas the ionization may cause losses. With a continuous flow the jet may be quite silent.

5.6 Plasma fusion chamber

As we have seen, high pressures and velocities can be obtained with the water explosions. Could this be used to initiate fusion processes? Hawke suggested that a mass impacting on a deuterium-filled pellet with a speed of 150 km/s would initiate fusion reactions [31]. In 1989 Beuhler et al. demonstrated that fusion reactions occur at impact speeds of about 100km/s. They used small deuterium clusters, which were easy to accelerate, and let them collide with deuterium-saturated titanium [6]. The situation is very different from thermonuclear fusion, which is governed by internal thermal collision processes[1]. Rambaut and Vigier have invented a method to accelerate heavy water with the electrodynamic explosions [58, 59].

Water is ejected into a chamber by ‘water-arc guns’, and collide in the center, where fusion reactions occurs, Figure 5.5A. In a similar chamber, Figure 5.5B, arcs can be discharged in the center of the chamber, making the situation resemble that of capillary fusion. The fusion process creates pressure and heat, which could be converted to useful energy. As the plasma need not to be uniformly heated to 5 million degrees, this may prove a more efficient way to release nuclear fusion energy than conventional hot fusion approaches.

[1] See Ref. [25] for a discussion on small scale impact fusion.

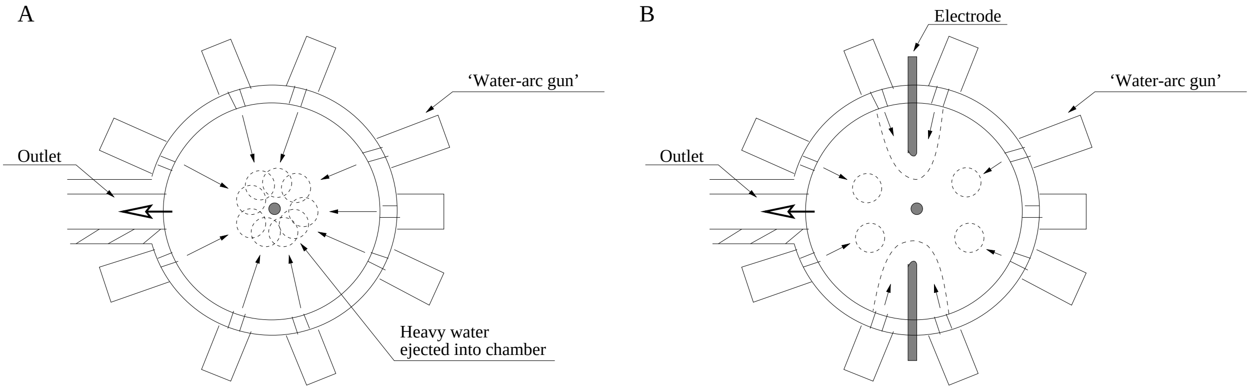

Figure 5.5: Plasma fusion chambers. (A) Impact fusion, (B) Impact fusion and arc-discharge fusion.

5.7 Capillary fusion

The longitudinal forces have been used in capillary fusion, a low energy fusion experiment invented by Lochte-Holtgreven et al. [42].

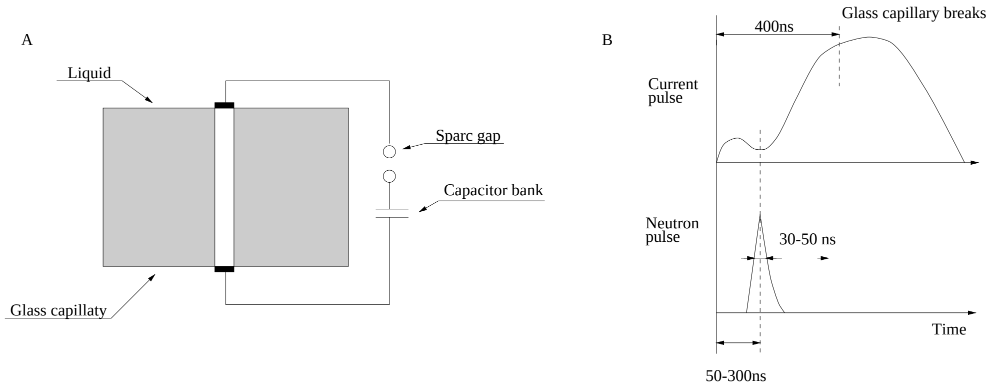

Figure 5.6: (A) The capillary fusion experiment. (B) Current pause and neutron burst.

A thin glass tube filled with e.g. $Li(ND_3)_4$ (lithium solved in heavy ammonia) is subjected to current pulses from a capacitor bank charged to 150-200kV typically, Figure 5.6A . After 50-200ns a drop in the current occurs due to longitudinal disintegration of the solution. During this drop, or 'current pause', a burst of $10^4 - 10^5$ neutrons is produced, Figure 5.6B. (With higher currents as many as 10^9 have been produced.) If the heavy ammonia ND_3 is substituted by light ammonia NH_3, no neutrons are produced.

The energy supplied in these experiments is about 500J, sufficient to heat the capillary uniformly to about 5000K, and probably less, as ionization energy isn't included in the calculation. Thus the neutrons cannot be produced by thermonuclear reactions; rather acceleration processes seem to be the candidate [37, 60].

As mentioned above, the wire disintegrates into beads during the current pause. In the cases where solid deuterium or deuteriated polyethylene is used the fragmentation resembles the 'wire striations' photographed by Nasiłowski [51, 24], see Figure 5.7.

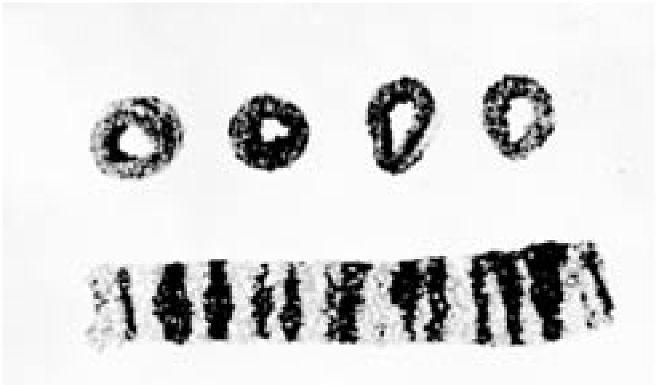

Figure 5.7: Striations produced by exploding wires. The dark bands are oxidized copper. From Jan Nasiłowski, Instytut Elektrotechniki, Warsaw, Poland.

Rambaut has proposed that the fusion reactions may be caused by quantum tunneling of deuterium. The presence of screening electron clouds would dilute the repulsion between the nuclei, making the probability of tunneling substantial [60, 61]. The situation is quite different from the nuclear reactions that occur in particle accelerators, where the nuclei collide head on in vacuum.

An advantage of this kind of fusion experiment is that it is relatively cheap and simple to carry out.

Chapter 6

Summary

In this report I have investigated how longitudinal electrodynamic forces (forces in the direction of current flow) manifest themselves in many experiments. When it comes to forces in solid and liquid metal conductors, the phenomena seem to be possible to explain by means of longitudinal Maxwell stresses, given e.g. by:

$$\sigma(r) = \frac{\mu_0 I^2 r^2}{8\pi^2 R^4} \ , r \leq R$$

This stress has its maximum at the periphery of the conductor, and is zero at the center. In addition to this a pinch pressure acts, which by hydraulic action causes a pressure at the center, in liquid conductors.

In dense plasmas, such as ionized water, the situation is more complicated. The forces that appear may be several magnitudes greater than those expected from electrodynamical considerations. Further research is needed. (It has been suggested that chemical bonding energy is released in the process.)

The Ampère electrodynamics approach advanced by Graneau has been analysed, and compared with the Maxwell stress approach. They turn out to be equivalent, one focusing on the charge carriers and the other on the field properties.

The stress in a conductor can be compared to that of magnets stacked side by side in a rectangular pipe. The stress predicted is of the same magnitude as that from the pinch forces, but is acting in different directions. Graneau's 'Finite current element' analysis (based on the Ampère electrodynamics) overestimates the forces, due to the mixing of uniform and non-uniform stress in that analysis.

The various 2:nd order relativistic theories involved have been discussed — the overview hopefully clears out some of the tangles. Forces from relativistic electric fields and from surface charges are minute in comparison to the Maxwell stress, but may be of importance for the charge distribution in plasmas.

The Maxwell stress approach is in good agreement with experimental data in e.g. the railgun and multiarc generator experiments. It is still unclear though if the forces are great enough to cause the fragmentation in Nasiłowski's experiment, although it seems as this may well be.

Several applications have been covered — in metal engineering, propulsion technology and electrodynamic fusion etc. Hopefully these applications show what could be achieved if the thinking is directed along new paths.

Appendix

Notation

The following notation is used throughout the report:

- Vectors are denoted by boldface fonts, as $\mathbf{r}$ and $d\mathbf{l}$. Unit vectors have a hat on top of them, as $\hat{\mathbf{r}}$.
- Scalar quantities are written with normal italic fonts, as I and μ_0. The same goes for the absolute length of a vector, i.e. $|\mathbf{r}| = r$
- r (which appears e.g. in the discussion on Maxwell stresses) means the radial distance from the center, in cylindrical coordinates. $\hat{\mathbf{r}}$ is the unit vector in the radial direction.
- Not to be confused with the above, $\mathbf{r}_{12}$ is the vector from point (current element) 1 to point (current element) 2; r_{12} is the length of this vector; and $\hat{\mathbf{r}}_{12}$ is the unit vector in its direction.
- r_0 is the distance from the origin. (It is used in the discussion on surface charges.)

A derivation of Grassmann's law

Biot-Savart's law, well known from magnetostatics, gives the magnetic field from a circuit:

$$\mathbf{H} = \frac{I_1}{4\pi} \oint \frac{d\mathbf{l}_1 \times \hat{\mathbf{r}}_{12}}{r_{12}^2}$$

$\mathbf{H}$ is the magnetic field; I_1 the electric current; $d\mathbf{l}_1$ is an infinitesimal section of the conductor; r_{12} the distance from $d\mathbf{l}_1$ to the point where the magnetic field is to be measured,; and $\hat{\mathbf{r}}_{12}$ the unit vector from $d\mathbf{l}_1$ to that point.

In differential form it becomes:

$$d\mathbf{H} = \frac{I_1}{4\pi} \frac{d\mathbf{l}_1 \times \hat{\mathbf{r}}_{12}}{r_{12}^2}$$

The magnetic force ($\mathbf{F}$) that moving charge experiences in a magnetic field is given by the Lorentz force law:

$$\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$$

Here q is the charge that is moving; $\mathbf{v}$ its velocity; and $\mathbf{B}$ the magnetic induction. With $\mathbf{B} = \mu_0 \mathbf{H}$ in vacuum (and air), and noting that $q_2 \cdot \mathbf{v}_2 = I_2 \cdot d\mathbf{l}_2$ we have:

$$d\mathbf{F} = I_2 d\mathbf{l}_2 \times \left(\frac{\mu_0 I_1}{4\pi} \frac{d\mathbf{l}_1 \times \hat{\mathbf{r}}_{12}}{r_{12}^2}\right) = \frac{\mu_0}{4\pi} \frac{I_1 I_2}{r_{12}^2} d\mathbf{l}_2 \times (d\mathbf{l}_1 \times \hat{\mathbf{r}}_{12})$$

which is Grassmann's law for the force between two current elements.

Bibliography

[1] H. Aspden *The Law of Electrodynamics* J. Franklin Inst., V.287, 1969, p.179-183

[2] Harold Aspden *The exploding wire phenomenon* Phys. Lett. A, V.107, 1985, p.238-40

[3] A.K.T. Assis *Deriving Ampère's law form Weber's law* Hadronic Journal, V.13, 1990, p.441-451

[4] H.W Baxter *Electric fuses* E.Arnold, London, 1950

[5] A.J.Bedford *Rail damage in small caliber railgun* I.E.E.E. Trans. Magn., V.20, 1984, p.348-55

[6] R.J. Beuhler, G. Friedlander, L. Friedman *Cluster-impact fusion* Phys. Rev. Lett., V.63,1989, p.1292-

[7] C. Blondel *Ampère et la creation de l'electrodynamique* Bibliotheque Nationale, Paris, 1982

[8] P.W. Bridgman *Breaking Tests under Hydrostatic Pressure and Conditions of Rupture* Phil. Mag., V.24, 1912, p.63-80

[9] C. Christodoulides *Comparison of the Ampère and Biot-Savart magnetostatic force laws in their line-current-element forms* Am. J. Phys., V.56, 1988, p.357-62

[10] Forrest F. Cleveland *Magnetic forces in a rectangular circuit* Phil.Mag., V.21, 1936, p.416-425

[11] Leon Dragone *Electric arc explosion: A thermal paradox* J. Appl. Phys., V.62, 1987, p.3477-3479

[12] Albert Einstein *The meaning of relativity* Princeton University Press, N.J., 1950

[13] Peter Graneau *Electromagnetic jet-propulsion in the direction of current flow* Nature V.295, 1982, p.311-312

[14] Peter Graneau *First indication of Ampère tension in solid electric conductors* Phys. Lett. A, V.97, 1983, p.253-55

[15] Peter Graneau *Ampère Tension in Electric Conductors* I.E.E.E. Trans. Magn., V.20, 1984, p.444-455

[16] Peter Graneau *Longitudinal magnet forces?* J. Appl. Phys., V.55, 1984, p.2598-2600

[17] Peter Graneau *Ampère-Neumann electrodynamics of metals* Hadronic Press, Mass., 1985

[18] Peter Graneau, Neal Graneau *Electrodynamic explosions in liquids* Appl. Phys. Lett., V.46, 1985, p.468-470

[19] Roy Azevedo, Peter Graneau, Charles Millet, Neal Graneau *Powerful water-plasma explosions* Phys. Lett.A, V.117, 1986, p.101-105

[20] Peter Graneau *Wire Explosions* Phys. Lett. A, V.120, 1987, p.77-79

[21] Peter Graneau *Amperian recoil and the efficiency of railguns* J.Appl.Phys., V.62, 1987, p.3006-3009

[22] Peter Graneau *The cause of thunder* J. Phys. D, V.22, 1989, p.1083-1094

[23] Peter Graneau *Electrodynamic seawater jet: An alternative to the propeller ?* IEEE Trans. Magn., V.25, 1989, p.3275-77

[24] Peter Graneau, Neal Graneau *The role of Ampère forces in nuclear fusion* Phys. Lett. A, V.165, p.1-13

[25] Peter Graneau *Heavy-Water-Arc Gun for Impact Fusion* Galilean Electrodynamics, July/August 1992, p.63-65

[26] Peter Graneau, Neal Graneau *Ampère force calculation for filament fusion experiments* Phys. Lett. A, 174, 1993, p.421-27

[27] Peter Graneau, Neal Graneau *Einstein versus Newton* Carlton Press, N.Y., 1993

[28] Peter Graneau, Neal Graneau *Newtonian electrodynamics* World Scientific, London, to be published: Summer 1996

[29] Peter Graneau *Solar Energy from Water* paper to be presented at the 4:th World Renewable Energy Congress, Denver, June 1996

[30] P. Hatzikonstantinou, P.G. Moyssides *On the radiation of the Electromagnetic Impulse Pendulum* Il Nuovo Cimento, V.13D, 1991, p.1093-1099

[31] R.S. Hawke *Device for launching 0.1g projectiles to 150km/s or more to initiate fusion* Atomenergie (AEKT), V.38, 1981, p.35-

[32] Mark A. Heald *Electric fields and charges in elementary circuits* Am. J. Phys., V.52, 1984, p.522-526

[33] Carl Hering *Electrodynamic forces in electric furnaces* Trans. Amer. Electrochemical Soc. v.39, 1921, p.313-30

[34] Carl Hering *Revision of some of the electromagnetic laws* J. Franklin Inst. V.194, 1921, p.599-622

[35] Carl Hering *Electromagnetic Forces; a Search for More Rational Fundamentals; a Proposed Revision of the Laws* Trans. of the A.I.E.E. V.42, 1923, p.311-40

[36] A.M. Hillas *Electromagnetic jet-propulsion: non-lorentzian forces on currents ?* Nature, V.302, 1983, p.271

[37] Sture K. Händel, Olle Jonsson *Capillary fusion in very dense plasmas* Atomkernenergie/Kerntechnik, V.36, 1980, p.170-72

[38] T. Ivezić *Electric fields from steady currents and unexplained electromagnetic experiments* Phys. Rev. A, V.44, 1991, p.2682-2685

[39] J.D. Jackson *Classical Electrodynamics* John Wiley & Sons, N.Y., 1975

[40] Oleg Jefimenko *Electricity and Magnetism* 2:nd ed., Electret Scientific Company, W.Virginia, 1989

[41] Allan N. Kaufman, Peter S. Rostler *The Darwin Model as a Tool for Electromagnetic Plasma Simulation* Physics of Fluids, V.14, 1971, p.446-448

[42] W. Lochte-Holtgreven ... Atomkernenergie, V.28, 1976, p.150-

[43] J.C. Maxwell *A treatise on electricity and magnetism* Clarendon Press, Oxford, 1873, §687

[44] J.C. Maxwell *Ibid.* §527

[45] Parry Moon, Domina E. Spencer *Electromagnetism without Magnetism: An Historical Sketch* Am. J. Phys., V.22, 1954, p.120-124

[46] Parry Moon, Domina E. Spencer *A new electrodynamics* J. Franklin. Inst., V.257, 1954, p.369-382

[47] Parry Moon, Domina E. Spencer *On the Ampère force* J. Franklin. Inst., V.260, 1955, p.295-311

[48] Parry Moon, Domina E. Spencer *On electromagnetic induction* J. Franklin. Inst., V.260, 1955, p.213-226

[49] Parry Moon, Domina E. Spencer *Some electromagnetic paradoxes* J. Franklin. Inst., V.260, 1955, p.373-396

[50] Paul G. Moyssides *Experimental Verification of the Biot-Savart-Lorentz and the Ampere Force Laws in a Closed Circuit, Revisited* I.E.E.E Trans. Magn., V.25, 1989, p.4298-4306, p.4307-4312, p.4313-4321 (3 Articles)

[51] Jan Nasiłowski *Phenomena Connected with the Disintegration of Conductors Overloaded by Short-Circuit Current* (in Polish) Przeglad Elektrotechniczny, 1961, p.397-403

[52] Jan Nasiłowski *Unduloids and striated Disintegration of Wires* Exploding Wires, W.G. Chase, H.K. Moore Eds., Vol.3, Plenum, N.Y., 1964

[53] Edwin F. Northrup *Some newly observed manifestations of forces in the interior of an electric conductor* Phys. Rev., V.24, 1907, p.474-97

[54] V. Peoglos *Measurement of the magnetostatic force of a current circuit on a part of itself* J. Phys. D, V. ,1988, p.1055-1061

[55] D.R. Peterson, C.M. Fowler, C.E. Cummings, J.F.Kerrisk, J.V.Parker, S.P. Marsh, D.F. Adams *Design and testing of high-pressure railguns* I.E.E.E. Trans. Magn., V.20,1984, p.252-55

[56] M. Rambaut, J.P Vigier *Ampère forces considered as a collective non-relativistic limit of the sum of all Lorentz interactions acting on individual current elements; possible consequences for electromagnetic discharge stability and tokamak behaviour* Phys. Lett. A, V.148, 1990, p.229-238

[57] M. Rambaut *Macroscopic non-relativistic Ampère EM interactions between current elements reflect the conducting electron accelerations by the ion's electric fields* Phys. Lett. A, V.154, 1991, p.210-214

[58] M. Rambaut J-P. Vigier *Process and device for producing fusion energy from a fusible material* (in French) Patent: WO 91/15016 (PCT/FR91/00225)

[59] M. Rambaut J-P. Vigier *Method and device for producing fusion energy from a fusible material* (in French) Patent: WO 91/16713 (PCT/FR91/00305)

[60] M. Rambaut *Capillary fusion through Coulomb barrier screening in turbulent processes generated by high intensity current pulses* Phys. Lett. A, V.163, 1992, p.335-242

[61] M.Rambaut *Double screened Coulomb barrier accounts for neutrons production in cluster and other fusion experiments* Phys. Lett. A, V.164, 1992, p.155-163, Erratum, V.165, 1992, p.480

[62] Linda J. Ruscak, R.N. Bruce *Multiarc Generator* I.E.E.E. Trans. Plasma Science, V.15, 1987, p.51-55

[63] P.G. Tait *Note on a modification of the apparatus employed for one of Ampère's fundamental experiments in electrodynamics* Phil. Mag. S.4, V.21, 1861, p.319-20

[64] J.G. Ternan *Stresses in rapidly heated wires* Phys. Lett. A, V.115, 1986, p.230-32

[65] R.A.R. Tricker *Early electrodynamics — the first law of circulation* Pergamon Press, Oxford, 1965

[66] F.T. Trouton, H.R. Noble *The Mechanical Forces Acting on a Charged Condenser Moving Through Space* Phil. Trans. R. Soc. London, V.202A, 1903, p.165-181

[67] E.T. Whittaker *A history of the theories of aether and electricity* Thomas Nelson, London, 1951