Filters and an Oscillator Using a New Solenoid Model

A transmission-line model enables a wider range of filter and oscillator designs

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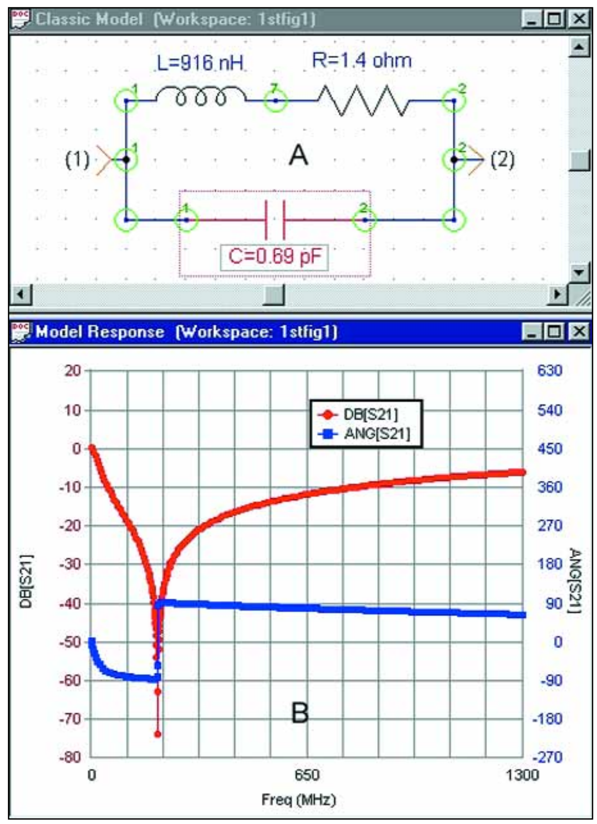
For most of us, discoveries in the art are rare. My books reviewed the contributions of many engineers and contained a limited number of original ideas. I recall many excited moments during my career that faded upon the realization that a "discovery" was actually a measurement error, misunderstanding or rediscovery.

In 1997, an engineer requested that I examine his measured data for an inductor. The data was unexpected, so I decided to measure a simple solenoid with a network analyzer. The analyzer display stunned me. This moment of excitement would not fade. The industry's accepted and century-old inductor model was wrong. Why had no engineer before me performed this simple experiment and written about it? The significance of the experiment began to unfold at my desk as I examined the problem mathematically. As is often the case, new knowledge is both satisfying and useful. The purpose of this paper is to describe new filter and oscillator structures suggested by the new model. First, I will review the historic inductor model, then the new one.



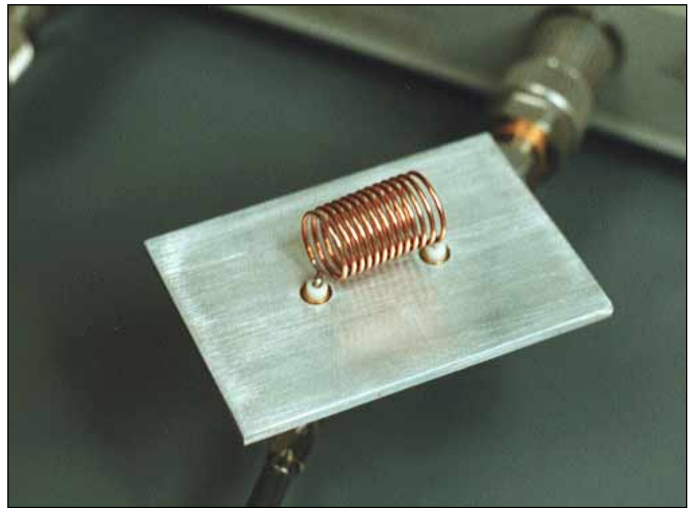
▲ **Figure 1. Classic inductor model with capacitance from Medhurst (a) and the transmission amplitude and phase responses (b).**

The historic model

One need only ponder a solenoid inductor to realize that the close spaced turns are capacitively coupled. But how do you measure capacitance that is shorted by a turn of wire? Webster [12] solved this problem by building a parallel-resonant mode oscillator using an inductor and its self-capacitance. Knowing the inductance (measured at low frequency) and the oscillating frequency, you may deduce the capacitance. From this capacitance, and series resistance from the wire loss, the model of the inductor given in Figure 1(a) was born. The transmission amplitude and phase versus frequency of this model that is simulated from the schematic and computed by the GENESYS [2] software program is given in Figure 1(b).

Numerous attempts at mathematically solving the self-capacitance failed. Medhurst [6] abandoned mathematical attempts in favor of an empirical approach with one end of the solenoid grounded. The accuracy of his capacitance is verifiable.

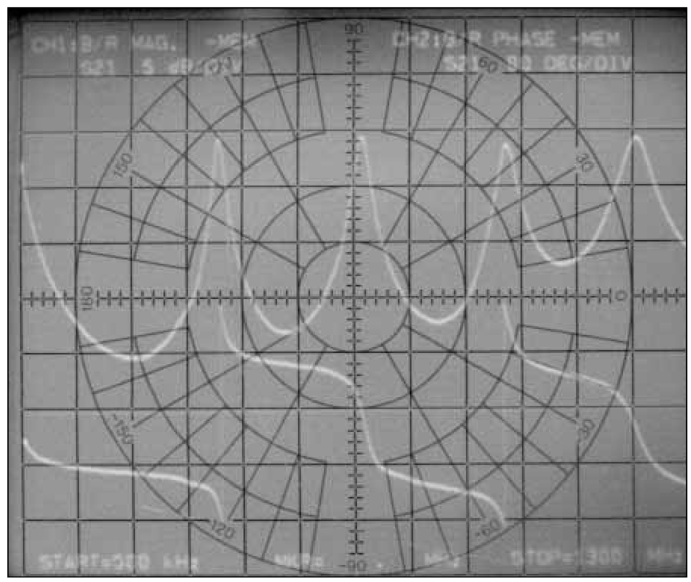
While Medhurst's capacitance is correct, the model he inherited is wrong. You may easily observe that the world is flat. It is equally obvious, and incorrect, that inter-winding capacitance is important. The phase shift from one turn to the next is small. With turns at nearly equal voltage potential phase, the effect of coupling capacitance is negligible. The capacitance between turns is effectively shorted. Since experiments showed that closer spacing did not increase the capacitance, it should have been suspected years ago that the capacitance is not turn-to-turn. Some rather esoteric explanations for this anomaly were promulgated. The capacitance that Medhurst quantified is capacitance of the solenoid to ground, not capacitance turn-to-turn. To understand how this error occurred, consider the analyzer data and a proposed new model.



▲ **Figure 2. The 12.9 turn copper-wire solenoid inductor mounted over a ground plane and configured for transmission amplitude and phase measurement.**

A new solenoid model

Given in Figure 2 is a 12.9 turn solenoid mounted over a ground plane [10]. It is wound with 18 gauge copper wire. The mean radius is 0.268 inches, the mean length is 0.890 inches, and the outside of the wire is 0.135 inches above the aluminum ground plane. The resulting transmission amplitude and phase are given in Figure 3. One striking feature is the periodic nature of the response, which clearly suggests a transmission-line model for the solenoid. This is not predicted by the classic model. Equally revealing is the transmission phase shift at the first anti-resonant mode of the classic model transitions from –90 to +90 degrees, while the measured data is continuous at –90 degrees. The classic model completely fails to predict the high-frequency behavior of the solenoid.



▲ **Figure 3. Transmission amplitude of the solenoid over ground (top) from 0.5 to 1300 MHz with a vertical scale of –40 to +10 dB and the transmission phase (bottom) with a scale of –270 to +630 degrees.**

Shown in Figure 4 are the transmission amplitude and phase of a transmission line with Z_0 = 796 ohms and an electrical length of 90 degrees at 195 MHz. Notice improved correlation to the measured solenoid responses. A transmission line may be modeled by distributed series inductance and shunt capacitance to ground. The characteristic impedance and electrical line length are related to the per unit series inductance and shunt capacitance by

$$Z_0 = \frac{L_0}{C_0} \text{ ohms} \tag{1}$$

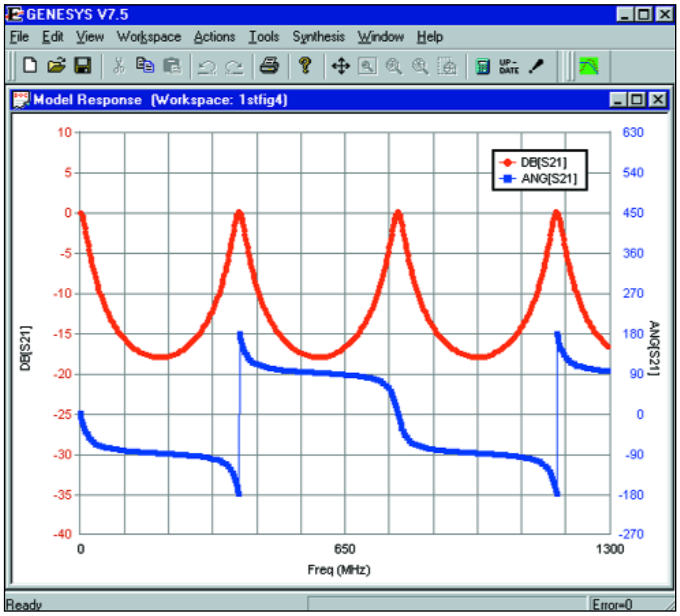
$$\theta = 2\pi F_0 \sqrt{L_0 C_0} \text{ radians} \tag{2}$$

or

$$\theta = 360 F_0 \sqrt{L_0 C_0} \tag{3}$$

This model predicts higher-order modes by static series inductance and shunt capacitance measured at low frequency. The model's elegance is further confirmed by the fact that these static parameters are easily calculated using existing mathematical techniques and that these techniques are equally applicable to diverse configurations, such as a solenoid above a ground plane or coax with a helical inner conductor. Mathematical calculation of the capacitance is now straightforward.

If the classic model assumed by Medhurst is wrong, how did he obtain the correct value of capacitance?



▲ **Figure 4. Transmission amplitude of the solenoid over ground (top) from 0.5 to 1300 MHz with a vertical scale of –40 to +10 dB and the transmission phase (bottom) with a scale of –270 to +630 degrees.**

Medhurst's parallel-mode resonator had one end of the inductor grounded. By chance, the inter-winding capacitance was effectively capacitance to ground.

Models for common configurations

The static inductance may be estimated using the popular formula from Wheeler [13],

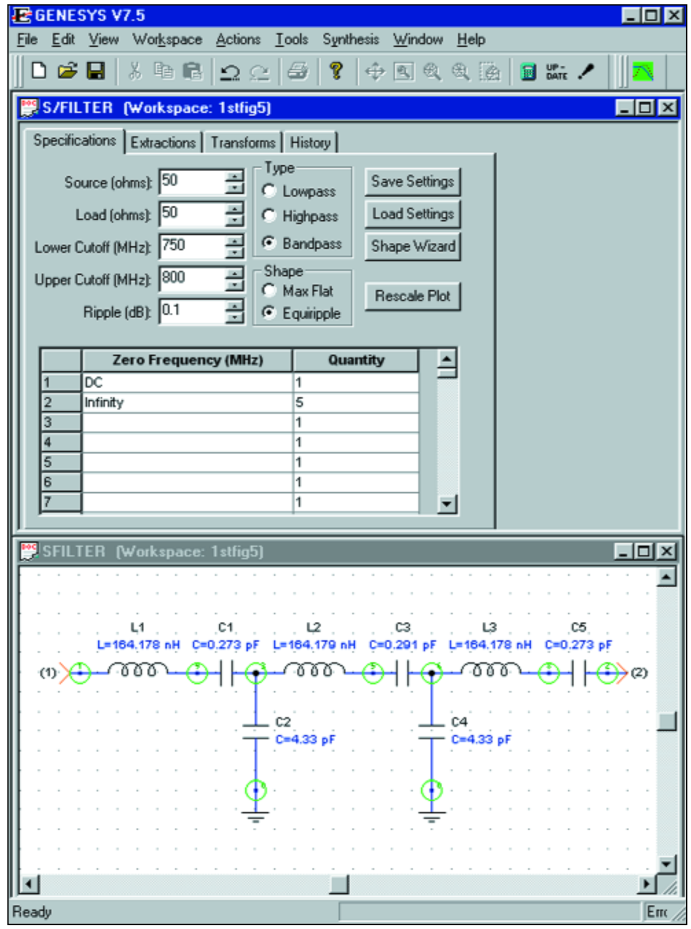
$$L = \frac{n^2 a^2}{9a + 10c} \text{ microhenries} \tag{4}$$

where n is the number of turns, a is the solenoid radius and c is the solenoid length. Wheeler's formula is accurate to ±1.5 percent for small wire diameter. For smaller gauge wire it overestimates the inductance. To compensate, I prefer to use the inside radius of the winding.

Table 1 gives the computed capacitance for common solenoid inductor configurations. The model transmission line impedance and electrical length are then computed from Equations (1) and (3) using these static inductance and capacitance. Alternatively, the static inductance and capacitance may be measured with low frequency instrumentation.

The capacitance is estimated assuming the solenoid is a solid cylinder. For example, to find the capacitance of the solenoid in Figure 2, the capacitance of a cylinder over ground is used. In Table 1, the characteristic impedance formula for various configurations [3] is used to compute the capacitance by the formula

$$C = \frac{c\sqrt{\varepsilon_r}}{V_0 Z_0} \text{ farads} \tag{5}$$



▲ **Figure 5. =S/FILTER= Specification tab (top) and schematic (bottom) for the 3rd order (6th degree) series-resonator bandpass filter.**

where V_0 is the velocity of light in a vacuum and Z_0 is the characteristic impedance of the configuration.

Effect of a shield on inductance

Wheeler's inductance formula assumes an unshielded solenoid. Bogle [1] gives an inductance reduction factor based on a conducting, non-magnetic, circular shield.

$$LF = 1 - \frac{\left(\dfrac{a}{a_s}\right)^2}{1 + \dfrac{1.55(a_s - a)}{c}} \tag{6}$$

The shielded inductance is found by multiplying the unshielded inductance by Bogle's factor. A shield radius of twice the solenoid radius and a solenoid length to radius ratio of 4 yields an inductance reduction of 18 percent. For square shields, a radius equal to 0.6 times the side dimension may be used. Shielding is ignored for a solenoid overground as data regarding the inductance reduction of a flat, adjacent, ground plane is unknown to this author.

The model below the 1st-resonant mode

For frequencies well below the 1st-anti-resonant mode, the capacitance is immaterial and the solenoid inductor is accurately modeled as a simple inductor. The reactance of a shorted transmission line is nearly linear (modeled by an inductor) for electrical line lengths up to λ/16 or 22.5 degrees. The frequency limit associated with this line length is

$$F_{\text{limit}} = \frac{1}{16\pi\sqrt{L_0 C_0}} \quad \text{hertz} \tag{7}$$

Above this frequency limit the new model is suggested.

Next, we will exploit the accuracy of this new model to create new classes of bandpass filters that use the series-resonant 2nd mode.

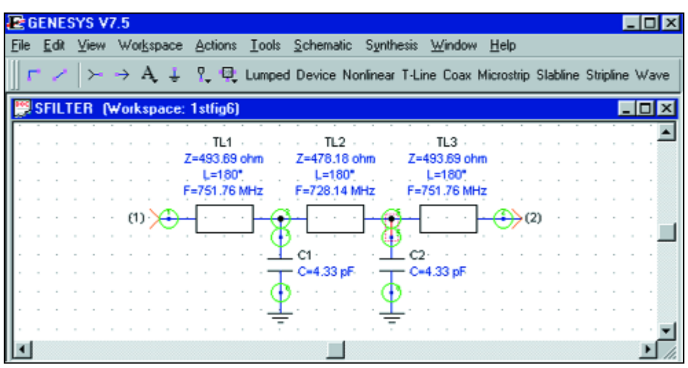
Capacitor-coupled 2nd-mode bandpass filter

Figure 5 gives the schematic of a third-order (sixth-degree) shunt capacitor-coupled, series-resonator lumped element Chebyshev 0.10 dB ripple, 750 MHz to 800 MHz bandpass filter. This filter is often designed using an approximate method described in Matthaei [5]. The accuracy of this routine degrades with increasing bandwidth. In this case, exact synthesis [11] was used to find element values. This filter has five transmission zeros at DC and one transmission zero at infinite frequency. An =S/FILTER= program screen, with the Specification tab active, is given at the top of Figure 5.

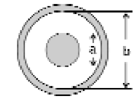
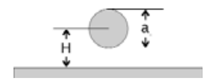
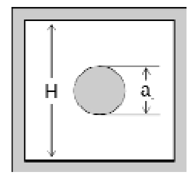
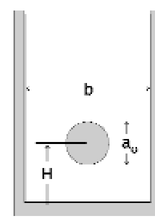
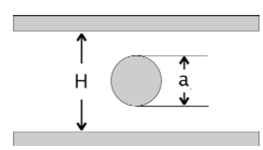
The L-C series resonators in Figure 5 may be replaced with equivalent transmission lines [8] that are 180 degrees long at the resonant frequency of that branch and with a characteristic impedance of

$$Z_0 = \frac{2\omega L}{\pi} = 4F_0 L \quad \text{ohms} \tag{8}$$

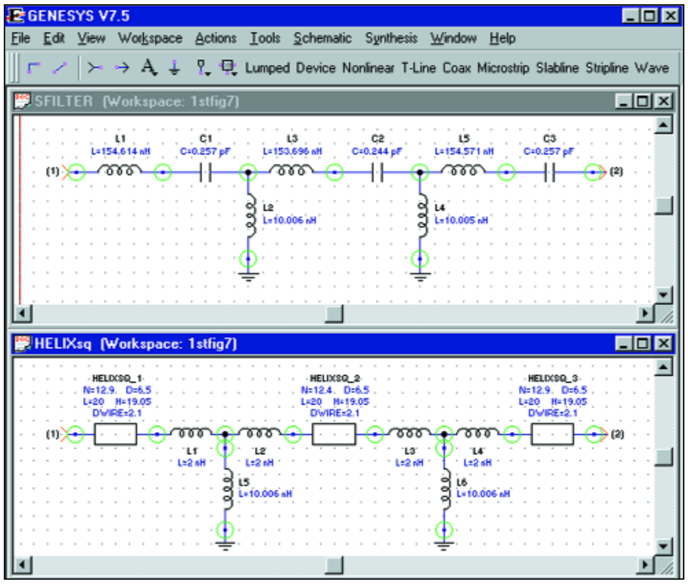
where L is the inductance in each respective branch. The inductor and capacitor in each series resonator



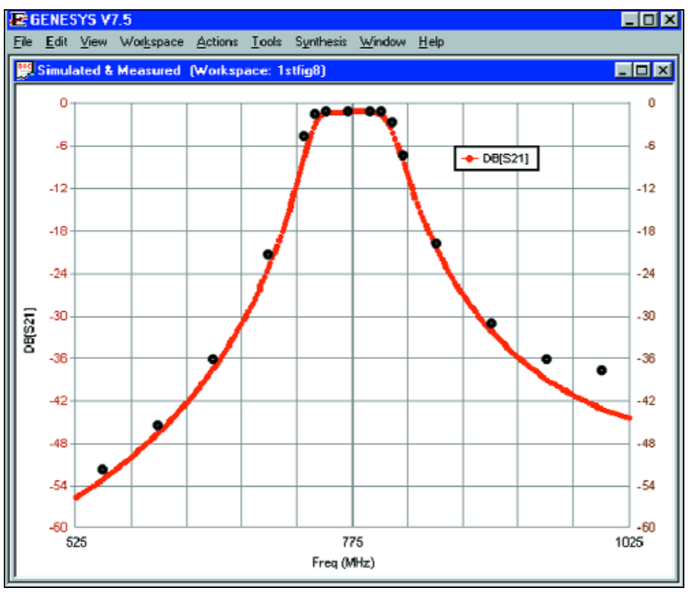
▲ **Figure 6. Shunt-capacitor coupled bandpass with series *L-C* resonators replaced with 2nd-mode solenoids modeled by transmission lines. Physical solenoids are designed using Equations (1) and (3) with inductance from Equation (4) and capacitance from Table 1.**

Cylindrical shield	 $C_0 = \dfrac{c\varepsilon_r}{59.952V_0 \ln\left(\dfrac{b}{a_0}\right)}\ pF$
Solenoid over plane	 $C_0 = \dfrac{c\varepsilon_r}{59.952V_0 \ln\left(\dfrac{2H}{a_0} + \sqrt{\left(\dfrac{2H}{a_0}\right)^2 - 1}\right)}$
Square shield	 $C_0 = \dfrac{c\varepsilon_r}{59.952V_0 \ln\dfrac{1.0787H}{a_0}}\ pF$
Trough shield	 $C_0 = \dfrac{c\varepsilon_r}{59.952V_0 \ln\left(\dfrac{4b}{\pi a_0}\tanh\dfrac{\pi H}{b}\right)}\ pF$
Slabline shield	 $C_0 = \dfrac{c\varepsilon_r}{59.952V_0\left(\ln\left(\dfrac{\sqrt{X}+\sqrt{Y}}{\sqrt{X-Y}}\right) - \dfrac{R^4}{30} + 0.01R^8\right)}$; $R = \dfrac{\pi a_0}{2H}$; $X = 1 + 2\sinh^2 R,\ Y = 1 - 2\sin^2 R$

▲ **Table 1. Static capacitance computed from the characteristic impedance of solid-cylinder models of solenoid configurations. $V_0 = 1.180285 \times 10^{10}$ for dimensions in inches and $V_0 = 2.997925 \times 10^{11}$ for dimensions in millimeters.**



▲ **Figure 7. Shunt-inductor coupled series-resonator bandpass filter (top) designed by =S/FILTER= and with resonators replaced with 2nd-mode solenoids (bottom). Series inductors model the wire connecting the solenoids. The helix parameters are for solenoids in a square shield.**



▲ **Figure 8. Simulated and measured amplitude response of the shunt-inductor coupled 2nd-mode solenoid bandpass.**

should not be confused with the static inductance and capacitance of the solenoids used to form the resonators. The schematic of the final shunt-capacitor coupled 2nd-mode bandpass is given in Figure 6. The solenoids are operated at the series-resonant 2nd mode and are depicted in Figure 6 as transmission lines.

The substitution of the series *L-C* resonators with solenoid transmission lines converts transmission zeros at infinite frequency to reentrant modes at frequencies above the passband. The filter in Figure 5(b) has no transmission zero at DC, which results in reduced low-frequency rejection.

2nd-mode bandpass filter using only solenoids

Figure 7 gives the schematic of a three-section series-resonator bandpass using coupling inductors rather than coupling capacitors. This structure has five transmission zeros at DC and one at infinite frequency. On the bottom in Figure 7 the series *L-C* resonators have been replaced with 2nd-mode solenoids.

Figure 8 gives the simulated response of the inductive-coupled 2nd-mode bandpass. A photograph of a prototype filter with solenoids wound using 18-gauge wire is given in Figure 9. Measured data is superimposed as circular points on the simulated response in Figure 8. The frequencies of the 2nd-mode resonators in the prototype filter were tuned using bendable metal tabs soldered to the side of the square shield. This introduces a small amount of capacitance, thus lowering and adjusting the frequency of each resonator.

Notice that this filter requires no capacitors: resonators are formed by the series-resonant 2nd-mode of the new solenoid model.

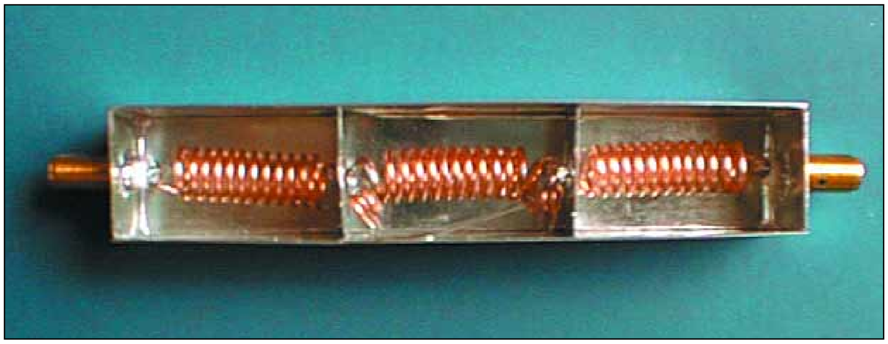
Solenoid unloaded *Q*

Component *Q* (unloaded *Q*) is defined as the ratio of stored to dissipated energy. More energy is stored in a larger magnetic field, and inductor *Q* can be increased by increasing the radius of the solenoid. Unfortunately, increasing radius also results in increased solenoid capacity. Classic inductor theory dictates that this capacitance must be much less than the inductor reactance. This limits the solenoid size and thus the available inductor *Q*.

Resonators constructed using the theory of the new model do not suffer this limitation. This capacity is a natural and desired element of the resonator. Therefore the resonator can be physically larger and can achieve higher unloaded *Q*.

My work with the new model has not yet specifically addressed unloaded *Q*. Nevertheless, the insertion loss of resonators and filters that have been constructed suggest that the unloaded *Q* predicted by Medhurst [6] is valid for the new transmission line model.

As solenoid size increases, radiation loss becomes more significant. This is typically not significant for inductors designed using the classic theory because inductor capacitance limits the maximum size. The new model permits larger solenoid size, and radiation is more likely to become significant. While solenoids enclosed by a square or round enclosure do not suffer from radiation loss, a solenoid over ground is more susceptible. The maximum size of shielded solenoids is limited by moding in the enclosure. Additionally, the size and therefore the unloaded *Q* is limited by the fact that the number of



▲ **Figure 9. Prototype 2nd-mode bandpass. 1.5 turn shunt-coupling inductors are barely visible at the inside of the end walls of the center section.**

turns to achieve resonance decreases with an increasing solenoid radius. As the number of turns decreases below a few turns, the phase shift between turns becomes significant and the model fails.

2nd-mode oscillator

The potential for improved unloaded Q of the new solenoid model supports its use in oscillators with lower phase noise. Examination of the benchmark paper by Leeson [4] reveals that neither a particular oscillator topology nor circuit complexity is required to achieve low phase noise. In fact, overly complex designs risk additional resonances from component and layout parasitics. Design elegance remedies these problems.

I started with the selection of a Mini-Circuits MAR-3 silicon MMIC for the sustaining amplifier. To illustrate the application of basic design elements toward design elegance, let me describe the thoughts that drove the development of the proposed oscillator in Figure 10. This design achieves a loaded Q of 59 using a 2nd-mode solenoid resonator.

- The natural input and output impedances of the MAR-3 are near 50 ohms. To achieve a loaded Q of 59 would require a characteristic impedance of nearly 4000 ohms in a series connected 2nd-mode solenoid. This requires high inductance and low capacitance resulting in small wire and poor unloaded Q, therefor eresonator coupling will be used.
- Supply voltage to the MAR-3 is typically delivered through the device output using a resistor or choke inductor. We will use this same inductor as a shunt-coupling element to effectively lower the impedance of the MAR-3 presented to the 1st-mode resonator, thus increasing loaded Q.
- A shunt-coupling reactor is also required at the output of the 2nd-mode resonator. Because the 2nd-mode solenoid is now at DC supply potential, a coupling capacitor is selected so as not to short the supply. A coupling inductor would require a bypass capacitor.
- To form oscillator feedback, the output of the resonator must be connected to the input of the MAR-3. This would short the supply voltage to the MAR-3 input. A coupling capacitor is required here.
- The gain, phase shift and input/output match of the amplifier-resonator cascade must be managed to satisfy Barkhousen's oscillation criteria. The values of all elements in the design are adjusted by computer optimization in GENESYS [2] to satisfy these criteria and to achieve the desired loaded Q.

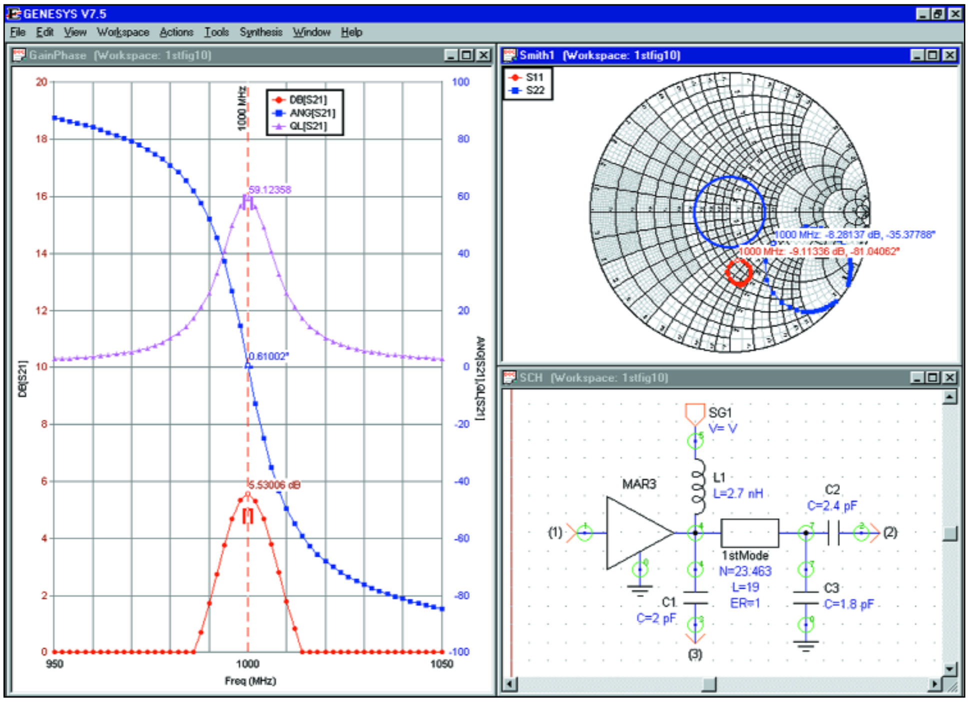
The open loop (from port 1 to port 2) gain and phase response of the design are given in Figure 10. The phase shift is near zero degrees at 1000 MHz, and the gain margin is 5.53 dB. To form the oscillator, the output is connected to the input. Since the oscillator is self-terminating, for the analysis to be accurate, the cascade input and output impedances should be approximately matched: in this case 8 dB or better return loss. The output power is taken through a 2 pF coupling capacitor at port 3.

It is not guaranteed that any chosen topology will satisfy all of these criteria simultaneously. In fact, the schematic in Figure 10 was not my first attempt for this design. Historically, topologies that satisfied design criteria were named after their discoverer, for example, Hartley. Design based on the fundamentals frees the designer from the shackles of a particular topology and often results in a more elegant, higher performance design. For a further description of design methods please refer to [9].

Limitations of the new model

This paper illustrates the usefulness of the new solenoid model. I found it compelling that inductance and capacitance measured at low frequency are capable of predicting solenoid behavior at high frequencies. However, notice that the resonances in the measured solenoid data in Figure 3 are not harmonically related as they would be for a simple transmission line model. The reason is that propagation on a solenoid over a ground plane is dispersive; it is not pure-TEM mode. Only one month after Rhea [10], Mezak [7] published a paper that presents a solution to the solenoid that considers dispersion. Mezak's solution is more mathematically involved than Rhea's solution, but Mezak's is more accurate.

Shielded solenoids are less dispersive and therefore the simpler model is more applicable. In Rhea [10], the measured data in Figure 16 of the helical coax unit shown in Figure 15 exhibits harmonically related 1st (anti-resonant), 2nd (series-resonant) and 3rd (anti-resonant) modes. Dispersion is also low for solenoids that are small with respect to a free-space wavelength and



▲ **Figure 10. The open loop transmission gain and phase and loaded *Q* (left) of a 1 GHz 2nd-mode solenoid oscillator (lower right). The cascade input and output return loss are given on the Smith chart (upper right).**

that are mounted close to ground planes. Dispersion is most severe in large solenoids with few turns, such as the unit in Figure 18 of Rhea [10].

References

1. A.G. Bogle, "The Effective Inductance and Resistance of Screened Coils," *Jour. IEEE,* 1940: 299.

2. *GENESYS 7 User's Guide,* Eagleware Corporation, Norcross, GA, 1999.

3. M.A.R. Gunston, *Microwave Transmission-Line Impedance Data,* Noble Publishing, Atlanta, 1996.

4. D.B. Leeson, "A Simple Model of Feedback Oscillator Noise Spectrum," *Proc. IEEE,* February 1966: 329–330.

5. G. Matthaei, L. Young and E.M.T. Jones, *Microwave Filters, Impedance-Matching Networks, and Coupling Structures,* Artech House, Dedham, Massachusetts, 1980.

6. R.G. Medhurst, "H.F. Resistance and Self-Capacitance of Single-Layer Solenoids," *Wireless Engineer,* February 1947: 35–43; and March 1947: 80–92.

7. J.A. Mezak, "Modeling Helical Air Coils for Wireless and RF Applications," *RF Design,* January 1998: 77–79.

8. R.W. Rhea, *HF Filter Design and Computer Simulation,* Noble Publishing, Atlanta, 1994.

9. R.W. Rhea, *Oscillator Design and Computer Simulation,* Noble Publishing, Atlanta, 1995.

10. R.W. Rhea, "A Multimode High-Frequency Inductor Model," *Applied Microwave & Wireless,* November/December 1997: 70–80.

11. =S/FILTER= Manual, Eagleware Corporation, Norcross, GA, 2000.

12. A.G. Webster, "An Experimental Determination of the Period of Electrical Oscillations," *Physical Review,* Vol. VI, 1898: 297–314.

13. H.A. Wheeler, "Inductance Formulas for Circular and Square Coils," *Proc. IEEE,* Vol. 75, No. 2, 1982: 256–257.

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