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Tesla Transformer Design and Application in Insulator Testing

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ABSTRACT

This paper presents an outline of the theory of the Tesla transformer. The circuit behaviour under the various conditions of coupling and tuning is investigated. The basic governing equation of the secondary output voltage of the device is analyzed to determine the constraint on the circuit parameters for achieving maximum voltage gain. To this end, new equations are derived. Also the construction of an experimental Tesla transformer is described and its practical application in high-voltage insulator testing is discussed.

1. INTRODUCTION

High-voltage impulse generators based on the Marx circuit design for charging capacitors in parallel and discharging them in series provide well-defined wave shapes for testing insulation structures over a range of different capacitances. Pulse transformer circuits, designed to operate as a Tesla or double-resonant circuit, can also be used for generating high alternating test voltages with the added advantages of low-cost, portability and high repetition rates of operation. Numerous papers have been published with particular emphasis on the use of the Tesla transformer in relativistic electron beam generators [1,2]. However, its application in the area of high voltage testing is limited due to the output waveform dependence on the load capacitance and the strong radio Inteference generated by the spark gap. When testing different test objects, the changing capacitive load affects the oscillation frequency of the secondary winding of the transformer. As a result, the primary circuit has to be re-tuned to maintain resonant operation.

Although it is difficult to control the waveshapes, the damped high frequency oscillations are somewhat similar to typical transient disturbances in power systems such as those caused by switching operations or by arcing grounds. Consequently, the Tesla transformer has been used as a supplement to the Marx generator in some HV laboratories for insulator testing. Recently, the Tesla transformer has been used successfully as part of a plant for synthetic testing of circuit breakers [3]. Its high repetition rate enables quick determination of the dielectric recovery curve in one operation.

After a brief review of the Tesla transformer theory, this paper discusses some of the more important aspects in its design and operation. Rather than optimizing the energy transfer efficiency, consideration is also given to other circuit designs for maximizing the output voltage where some new results are presented. As an example, the details of a prototype Tesla transformer with an output of a few hundred kV are given. Practical problems involved in the construction and its use in the flashover testing of some high voltage insulators are discussed.

2. THEORY OF OPERATION

2.1 Equation for the secondary output voltage:

A simplified equivalent circuit of the Tesla transformer is shown in Fig.1. Basically it consists of two LC circuits loosely coupled (air-core) through the mutual inductance M. Commutation across the spark-gap is initiated when the primary capacitor C_1, charged from an ac or dc source (not shown in the diagram), reaches a given potential. This results in an oscillatory current flowing in the primary circuit L_1C_1 which then induces oscillations in the secondary circuit L_2C_2. In practice, the output voltage is a high-frequency damped oscillation due to the effect of finite winding losses.

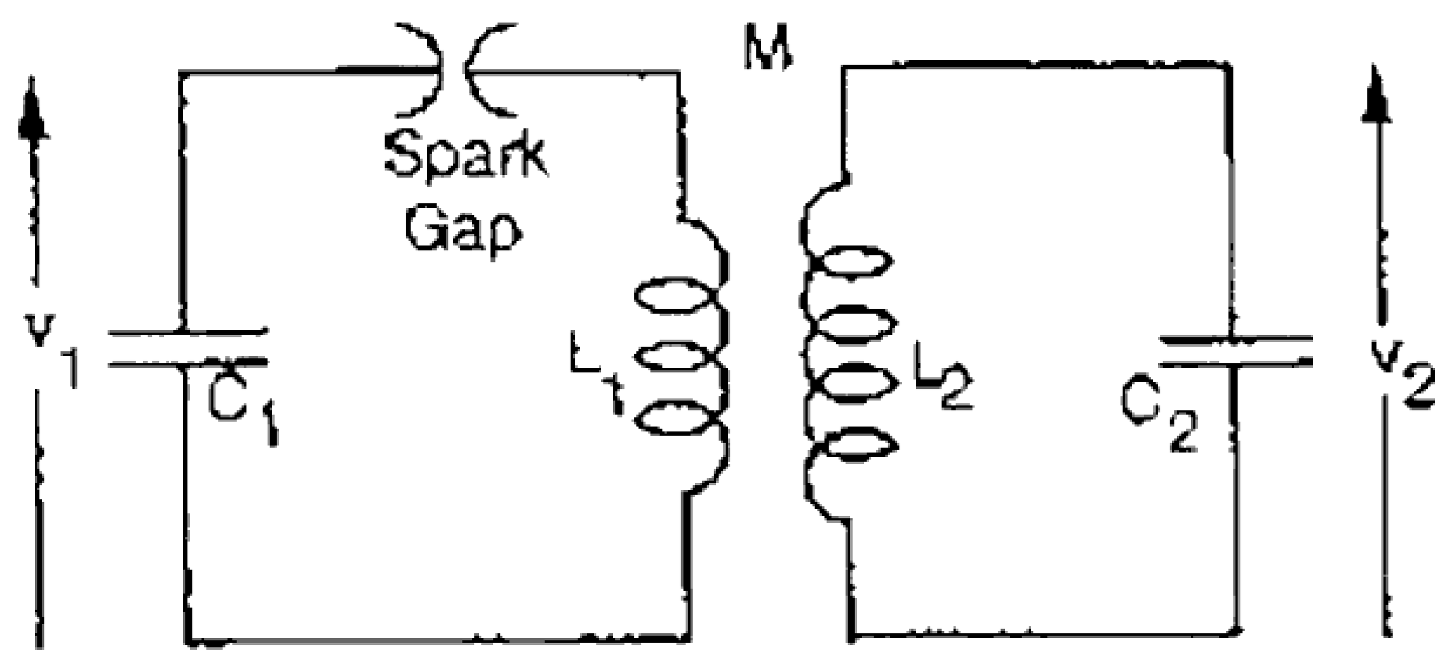

Fig.1: Basic circuit diagram of the Tesla transformer.

Assuming that the winding resistances are negligible, the circuit satisfies the following system of two linear homogeneous equations:

$$L_1\frac{di_1}{dt} + M\frac{di_2}{dt} + \frac{1}{C_1}\int i_1 dt = 0 \qquad (1a)$$

$$L_2\frac{di_2}{dt} + M\frac{di_1}{dt} + \frac{1}{C_2}\int i_2 dt = 0 \qquad (1b)$$

The auxiliary equation can be found by substituting $i_1 = C_1 (dv_1/dt)$ and $i_2 = C_2 (dv_2/dt)$ into the above equations and then eliminating v_1:

$$(1-k^2)\frac{d^4v_2}{dt^4} + (\omega_1^2 + \omega_2^2)\frac{d^2v_2}{dt^2} + \omega_1^2\omega_2^2 v_2 = 0 \qquad (2)$$

Here k is the coupling coefficient:

$$k = M/\sqrt{L_1L_2} \qquad (3)$$

which must satisfy the physical constraint $0<k<1$. ω_1 and ω_2 are the angular resonant frequencies of the uncoupled primary and secondary circuits respectively (also called the open-circuit resonances):

$$\omega_i = \frac{1}{\sqrt{L_iC_i}} \qquad i = 1,2 \qquad (4)$$

Eq.2 is a fourth-order linear homogeneous differential equation. It is a lengthy but straightforward process to find the analytical solution for the secondary output voltage $v_2(t)$. The full derivation can be found in a number of early references, e.g. [4].

It is of interest to determine how the output voltage changes with respect to the coupling coefficient k and the tuning ratio T, the latter being defined as:

$$T = (L_2C_2)/(L_1C_1) = \omega_1^2/\omega_2^2 \quad (5)$$

T must be positive to have any physical significance. The governing equation for the output voltage [4] can be rearranged in another form:

$$v_2(t) = \frac{2kV_1}{\sqrt{(1-T)^2+4k^2T}}\sqrt{\frac{L_2}{L_1}}\,\sin\left(\frac{w_1+w_2}{2}t\right)\sin\left(\frac{w_2-w_1}{2}t\right) \quad (6)$$

where V_1 is the initial voltage (at time t=0) across C_1. w_i's are the resonances of the primary and secondary circuits when coupled:

$$w_{2,1} = \omega_2\left[\frac{(1+T)\pm\sqrt{(1-T)^2+4k^2T}}{2(1-k^2)}\right]^{1/2} \quad (7)$$

The physical constraints on the possible values of k and T ensure that the arguments in the two square roots required in Eq.7 are always positive and hence w_i's are real. Also if only positive roots are used then:

$$w_2 > w_1 > 0 \quad (8)$$

Eq.6 shows that the output voltage is a high frequency oscillation $[(w_1+w_2)/2]$ which is amplitude modulated by another low frequency oscillation $[(w_2-w_1)/2]$.

2.2 Maximum voltage gain:

From Eq.6, the maximum voltage gain is:

$$G = \sqrt{\frac{1}{\left(\frac{1-T}{2k}\right)^2 + T}}\; G_L \qquad \text{where } G_L = \sqrt{\frac{L_2}{L_1}} \quad (9)$$

Or using Eq.5, another equivalent expression for G is:

$$G = \sqrt{\frac{T}{\left(\frac{1-T}{2k}\right)^2 + T}} \cdot G_C \qquad \text{where } G_C = \sqrt{\frac{C_1}{C_2}} \quad (10)$$

Eq.9 shows that for any value of T, an upper limit can be found by choosing the maximum value of k. With k approaching 1, the first term in Eq.9 becomes:

$$G_1 = \frac{2}{1+T} \quad (11)$$

Except for the constant multiplying factor G_L, this gives a theoretical upper limit on the voltage gain as a function of the tuning ratio. The actual maximum voltage gain is bound by this upper limit and zero. The problem now is to determine the proper combination of T and k so that the voltage gain is as close as possible to the limit set by Eq.11.

The constraint on T and k comes from Eq.6 where both the sine terms are required to be equal to ±1 simultaneously so that the gain given by Eq.9 can be achieved, i.e.

$$\frac{w_1+w_2}{2}t = \frac{\pi}{2}+m\pi \quad \text{and} \quad \frac{w_2-w_1}{2}t = \frac{\pi}{2}+n\pi \quad (12)$$

where m and n are integers (positive or negative). Without loss of generality, n can be set to zero (simply by shifting the time origin). Therefore:

$$\frac{w_2}{w_1} = \frac{1+m}{m} \quad (13)$$

Substituting w_i's as given by Eq.7 into Eq.13:

$$k = \sqrt{\frac{\alpha^2(1+T)^2-(1-T)^2}{4T}} \quad (14)$$

where:

$$\alpha = \frac{1+2m}{1+2m+2m^2} \quad (15)$$

Finally, use Eq.14 to eliminate k from Eq.9:

$$G = \sqrt{\frac{\alpha^2(1+T)^2-(1-T)^2}{T\alpha^2(1+T)^2}}\; G_L \quad (16)$$

Now, Eqs.8 and 13 show that m can be any integer excluding 0 and -1. Furthermore, Eq.15 shows that α is a function of m and that $\alpha(m) = -\alpha[-(m+1)]$. For example, $\alpha(2) = 5/13$ whereas $\alpha(-3) = -5/13$. In other words, α^2 has the same value for both m and -(m+1). Consequently, it is only necessary for the analysis to be carried out for the case of positive integers, i.e. m = 1,2,3,...

Eq.16 is a new result and is important in that it allows optimization of the gain to be easily achieved. For a given T, it is not difficult to see that the gain is maximized if m=1. On the other hand, for a given value of m Eq.16 has to be solved numerically and the maximum gain can be found from the plot of the gain versus T. Also if m is known, Eq.14 can be used to find the appropriate coupling coefficient. Table 1 shows the results for the first few positive integers m:

m	1	2	3	4	5	10
T	0.541	0.766	0.863	0.912	0.939	0.982
k	0.546	0.364	0.271	0.215	0.178	0.095
G/G_L	1.180	1.073	1.039	1.024	1.016	1.005

Table 1: Maximum gain ratios as a function of m and T.

It can be seen that as m increases the maximum of the ratio (G/G_L) decreases, approaching unity. Thus the maximum possible voltage gain occurs when k=0.546 and T=0.541 which gives:

$$G = 1.18\,G_L \quad (17)$$

This observation has also been noted elsewhere [5]. However, the analysis outlined here is more general whereas in that study it was carried out for only one special case where $w_2/w_1=2$ which is equivalent to m=1.

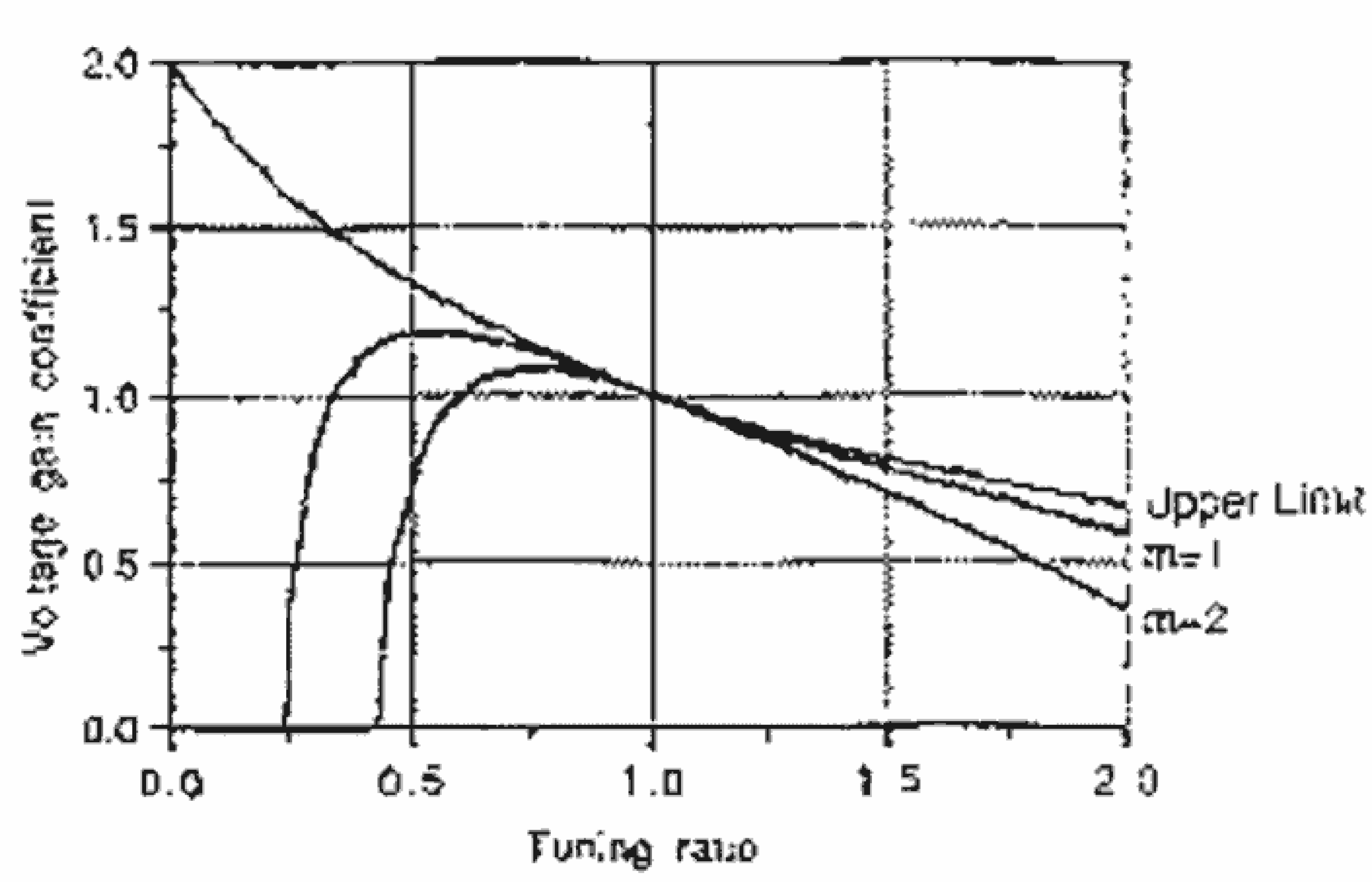

Fig.2: Plot of voltage gain vs tuning ratio.

The voltage gain coefficient versus the tuning ratio is plotted in Fig.2. Also shown is the upper limit as given by Eq.11. Note that for each value of m, α is fixed and the condition $0<k<1$, together with Eq.14, determines the range of valid tuning ratios:

$$\left[\frac{m}{1+m}\right]^2 < T < \left[\frac{1+m}{m}\right]^2 \quad (18)$$

Outside this range, it is not possible to solve Eq.16 so that the gain in this case is zero. To find a better gain, perhaps one needs to rely on a different analysis where it is not required to have the sine terms of Eq.6 equal to ±1 simultaneously. Another conclusion which can be drawn from Fig.2 is that for 0.5<T<1.5, the circuit parameters can be chosen to achieve a voltage gain very close to the theoretical upper limit.

The results from Table 1 show that regardless of how loose the coupling is, high voltage excitation is still possible provided the coils are properly tuned to the correct value. This explains the interesting phenomenon sometimes observed where other coils in the vicinity can respond with sparking even though they are not physically connected to the Tesla transformer. However as k decreases, higher harmonics are required to maximize the output voltage. Consequently the allowable tuning range, as determined by Eq.18, becomes tighter and thus the possibility of excitation is much reduced.

2.3 Maximum power transfer:

Consider the special case where the open-circuit resonant frequencies of the primary and secondary circuits are equal, i.e. $\omega_1 = \omega_2 = \omega$. This gives a unity tuning ratio (T=1). Eqs.6 and 7 reduce to:

$$v_2(t) = V_1 \sqrt{\frac{L_2}{L_1}} \sin(\frac{W_1+W_2}{2}t) \sin(\frac{W_2-W_1}{2}t) \qquad (19)$$

where:

$$W_{1,2} = \frac{\omega}{\sqrt{1 \pm k}} \qquad (20)$$

Obviously, the maximum voltage gain in this case is:

$$G = \sqrt{\frac{L_2}{L_1}} = \sqrt{\frac{C_1}{C_2}} \qquad (21)$$

which is not dependent on m. In other words, all the curves in Fig.2 coincide at T=1. Using Eq.14, the coupling coefficient becomes:

$$k = \alpha \qquad (22)$$

The above relationship is different from that derived in [6] which gives k = (2m-1)/(2m+1) where m=1,2,3,... It is only for the case of m=1 that both expressions give the same answer, i.e. k=0.6.

Usually, Tesla transformers are designed such that T=1 and k=0.6. Under this condition, total transfer of energy from the primary to the secondary capacitor is achieved [6]. However, the maximum possible voltage gain in this case is less than the maximum gain achieved with T=0.541 and k=0.546. Note that from Eq.20, the closed-circuit resonances have a frequency ratio of 2:1.

3. EXPERIMENT AND DISCUSSION

Although the basic Tesla transformer circuit is simple and requires very few components, there are practical problems that must be considered in the construction. Some useful suggestions can be found in reference [8]. It provides a collection of different circuit configurations and also practical design details of the components.

A small prototype Tesla transformer was contructed for use in the laboratory. The details of the design are as follows. A 50Hz ac supply is used. The oil-filled capacitor on the primary side is C_1=0.015μF and rated at 15kV; i.e. the maximum stored energy is ~1.7J. It is charged through a step-up 240V/15kV center-tap neon transformer, rated at 450VA. The spark gap is a rotary type which consists of a motor-driven bakelite disk with 8 metal studs equally spaced around the periphery. The discharge rate is dependent on the setting of the motor speed. Note that with a static spark gap, quenching of the spark gap is a common problem. Due to intensive heat and oxidation, electrode replacement and frequent adjustment of the gap distance are necessary. On the other hand, arc interruption is quicker with the rotary type and more controllable, thus allowing a higher firing rate. Also, the electrodes are subject to less wear because of the reduced arcing.

The former for the secondary coil is a length of PVC tubing. The diameter is d=11cm and the length is h=75cm. Its inductance L_2 can be calculated using the formula for self-inductance of an air-cored solenoid:

$$L = \mu_0 n^2 hA \qquad (23)$$

where n is the number of turns per metre, A is its cross-sectional area and μ_0 is the free-space permeability. With n=1700 turns/metre, L_2 is ~26mH (22mH measured with an LCR bridge).

The primary coil, using thicker gauge wire, is a cylindrical spiral pancake. The radii of the top and bottom sides are 30cm and 20cm respectively and the height is h=20cm. The total number of turns is N=10 and a number of tap points are provided to allow fine tuning. If the coil is considered as a cylinder with a mean radius r=25cm so that Eq.20 is applicable, its inductance is ~123μH. At the other extreme, if it is approximated as a flat or single-layer spiral pancake then the empirical formula [8,9] gives an inductance of ~83μH which is closer to the measured value of L_1=79μH.

For high frequency operation, the physical size of the secondary coil becomes significant, i.e. the total wire length becomes comparable to the wavelength of the oscillating signal. If it is tuned to function as a quarter-wave resonator then the voltage at one end of the coil is always fixed at the minimum (node) whereas the voltage at the other end can swing to the maximum (antinode). Usually the bottom end is made a node (by grounding it) and provided the primary coil is placed near this end, the problem of insulating the HV secondary coil from the lower voltage primary coil is much reduced.

Now, the total wire length of the secondary coil is [nh(πd)]. If the quarter wavelength theory is applied, the desired resonant frequency f_2 is:

$$f_2 = c / 4nh\pi d \qquad (24)$$

where c is the propagation velocity of electromagnetic waves in free space. For the coil above, f_2 is ~170kHz. Another factor attributed to the coil size is the distributed capacitance. It is assumed that this can be represented as a lumped capacitance C_d. If the capacitance of the test object is denoted as C_L then the total capacitance on the secondary circuit is the parallel combination of these two capacitors, i.e.:

$$C_2 = C_d + C_L \qquad (25)$$

By measuring the resonant frequency of the stand-alone secondary coil and using Eq.4, C_d can be determined which, in this case, is ~15pF. If C_L is much greater than C_d then the capacitive load of the test object has a direct impact on the output voltage waveshape. The effect of varying load capacitances on the maximum gain can be easily seen from Eq.10, e.g. doubling the load capacitance will reduce the gain by a factor of $\sqrt{2}$. On the other hand, if the load capacitance is small, the effect of the internal capacitance of the secondary winding becomes significant and must be taken into consideration. In this case, the circuit efficiency is reduced. However, the basic circuit of Fig.2 can be modified by adding an external inductance between the secondary output and the test object so that it operates as a triple-resonant transformer [7]. Depending on the selection of circuit parameters, complete energy transfer from the primary to the load can be achieved with the new circuit.

A small number of porcelain HV insulators are available for experiments. Their capacitances C_L vary from ~10pF to ~30pF, i.e. the same order of magnitude as C_d. To reduce the circuit sensitivity to the changing load, a small brass dome is added to the HV end of the secondary which increases the effective value of C_d to 36pF.

With C_2, L_2 and C_1 fixed, the remaining circuit parameters that can be adjusted to optimize the output voltage are L_1 and the coupling coefficient k. One possible procedure is first to select the tuning ratio T. A value around 1 is a good choice, the reason has been discussed before. This in turn determines the primary inductance L_1 and the coupling coefficient k is given by Eq.14. Note that k can be easily adjusted and measured based on the relationship [9]:

$$k = \sqrt{1 - (w_{oc}/w_{sc})^2} \qquad (26)$$

where w_{oc} and w_{sc} are the resonant frequencies of the secondary winding when the primary winding is open-circuited and short-circuited respectively. Adjustment is done by changing the proximity of the two windings.

The design for maximum power transfer operation is also straightforward. For example, take the case where C_L=10pF. Eq.25 gives C_2=46pF and Eq.5 determines the choice of L_1 which is ~67μH. From Eq.4, the open-circuit resonant frequency is 158kHz and then from Eq.20, the closed-circuit resonant frequencies are 250kHz and 125kHz. The maximum voltage gain is 18. If the peak charging voltage on the primary is 15kV, this corresponds to a maximum output of 270kV.

4. CONCLUSIONS

A brief review of the Tesla transformer theory was given. Based on the standard expression for the output voltage, the circuit behaviour with respect to the effect of coupling and tuning is analyzed to determine to condition for achieving maximum voltage gain. Design details of a small Tesla transformer are described. Preliminary flash-over tests have been made on some HV insulators. Although the output waveform is non-standard, such tests provide some useful indications of the effect on insulators caused by switching transient distubances in power systems. To date, very few experimental results have been obtained due to the difficulty in measuring the output voltage. The use of a capacitive divider will reduce the output voltage substantially and a much larger primary capacitor is required to compensate for the loss in gain. Work is in progress to devise an alternative measurement technique.

5. REFERENCES

/1/ C.R.J. Hoffmann, "A Tesla transformer high-voltage generator", Rev. Sci. Instrum., Vol.46, No.1, January 1975, pp.1-4.

/2/ H. Matsuzawa and S. Suganomata, "Design charts for Tesla-transformer-type relativistic electron beam generators", Rev. Sci. Instrum., Vol.53, No.5, May 1982, pp.694-696.

/3/ G.C. Damstra and J.A.J. Pettinga, "A six pulse kV Tesla transformer", Paper 62.13, 5th ISH, Braunschweig, Federal Republic of Germany, 1987.

/4/ J.D. Craggs and J.M. Meek, "High voltage laboratory techniques", Butterworths, London 1954, Chapter 3.

/5/ J.L. Reed, "Greater voltage gain for Tesla-transformer accelerators", Rev. Sci. Intrum., Vol.59, No.10, October 1988, pp.2300-2301.

/6/ D. Finkelstein, P. Goldberg and J. Schuchatowitz, "High voltage impulse systems", Rev. Sci. Instrum., Vol.37, No.2, February 1966, pp.159-162.

/7/ F.M. Bienniosek, "Triple resonant pulse transformer circuit", Rev. Sci. Instrum., Vol.61, No.?, June 1990, pp.1717-1719.

/8/ R.A. Ford, "Tesla coil secrets: construction notes and novel uses", Lindsay Publications, 1985.

/9/ A. Hund, "High-frequency measurements", McGraw-Hill, 2nd. Ed., 1951.

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