A First Technical Look at Witricity

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Rev 0.2

Recently, a group and MIT has introduced a technology they call “WiTricity”.

http://www.sciencemag.org/cgi/data/1143254/DC1/1

http://www.sciencemag.org/cgi/content/abstract/1143254

Now there is sufficient data on the system to analyze it with standard Tesla coiling techniques. With some small number rounding, we can assume a very similar situation using Tesla coils:

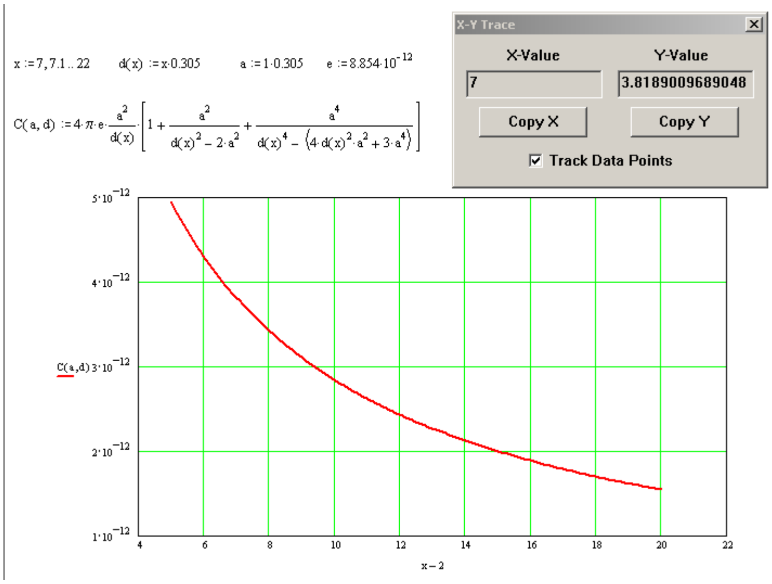
1. Two identical Tesla coils.
2. Fo = 10MHz.
3. Two spherical top terminals with a radius of 1 foot.
4. The distance between the sphere centers is 9 feet.
5. The primary is a single turn of 0.118 inch diameter wire with a radius of 1 foot.
6. The secondary is 5.25 turns 0.118 inch diameter wire with a radius of 1 foot and a height of 7.9 inches.
7. The primary to secondary spacing is 1.5 inches.

In this case, we will assume the secondary and primary coils are bound inside the spheres. Thus, coils that MIT uses are modeled as spheres instead of coils. From an electrostatic point of view, that is not far off from the actual situation but it greatly simplifies the models. In effect, two spheres could be used instead of the coils with sufficient electronics inside each to form the same drive voltage at the surface of the driving sphere and transform the received drive current at the receiving sphere. From an electrostatic wireless power transfer point of view, the shape and electronics behind the transmitter and receiver does not matter.

Two - 2 foot diameter spheres separated by a distance of say 7 feet can be estimated at 4pF from equation A.14 in the link below.

http://www.iue.tuwien.ac.at/phd/wasshuber/node77.html

The capacitance between the spheres can also be estimated at any distance.



Capacitance between the spheres vs. distance.

The standard Tesla coiling program MandK can calculate the coil parameters.

http://hot-streamer.com/TeslaCoils/Programs/MANDKV31.PDF
http://hot-streamer.com/TeslaCoils/Programs/MANDKV31.ZIP

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Mutual Inductance Program V3.1
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Solenoidal Primary Coil Geometry
Primary coil inside diameter (inches)=     24.000
Primary coil outside diameter (inches)=     24.000
Number of primary coil turns =      1.000
Last turn elevation in inches =      1.000
Wire diameter (inches)=      .1180

Solenoidal Secondary Coil Geometry
Secondary coil diameter (inches)=    24.000
Secondary coil height (inches)=     7.900
Number of secondary coil turns =     5.250
Secondary coil wire diameter (inches)=      .1180

Calculational Results
Primary coil inductance in microhenries:  Lp =            1.56
Secondary coil inductance in microhenries:  Ls =            21.48
Figure of Merit (square root of Ls/Lp) =      3.71
Primary coil wire length in feet =      6.28
Secondary coil wire length in feet =      32.99
DC secondary resistance in ohms:       .03
Secondary coil distributed capacitance in picofarads:     35.51
 (Medhurst formula, assumes one end of secondary is grounded)

Mutual Inductance Results
Position is the secondary coil bottom wire position in inches
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above the bottom wire of the primary coil.
A negative value means the bottom wire of the secondary is below the bottom wire of the primary.
A positive value means that the bottom wire of the secondary is above the bottom wire of the primary coil.
M = Mutual Inductance in microhenries
K = Coefficient of Coupling: K = M / square root (Lp x Ls)

Position	M	K
.000	3.680	.6364
.500	3.011	.5207
1.000	2.645	.4574
1.500	2.373	.4103
2.000	2.152	.3721

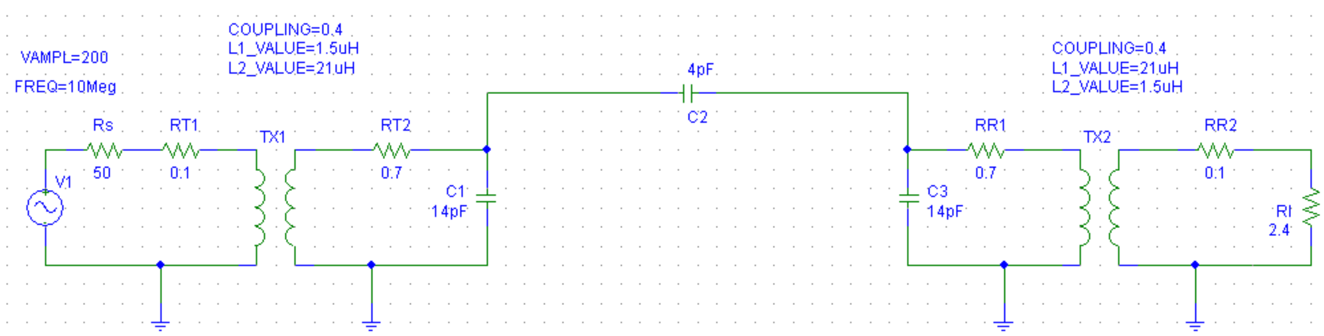
Operating Characteristics
Ctoroid is the added capacitance of the top terminal.
Fres is the resonant operating frequency in kilohertz.
Cpri is the required primary capacitance in nanofarads.
Zpri is the primary surge impedance in ohms.
Zsec is the secondary surge impedance in ohms.
Qsec is the secondary coil quality factor Q.
Rac is the secondary coil RF resistance in ohms.

Ctoroid(pF)	.0	5.0	10.0	15.0	20.0	25.0	30.0	35.0
Fres(kHz)	5763.2	5395.8	5090.8	4832.2	4609.5	4414.9	4243.1	4089.9
Cpri (nF)	.49	.56	.63	.70	.77	.83	.90	.97
Zpri(ohms)	56.4	52.8	49.8	47.3	45.1	43.2	41.5	40.0
Zsec(ohms)	778.	728.	687.	652.	622.	596.	573.	552.
Qsec	1096.0	1060.2	1029.6	1002.8	979.1	958.0	938.9	921.6
Rac (ohms)	.7	.7	.7	.7	.6	.6	.6	.6

The Key Parameters are:

Primary inductance = 1.56uH
Secondary inductance = 21.48uH
Coupling at 1.5 inches = 4.103

Rounding the numbers a bit we can put all this data into a dual coupled MicroSim Tesla coil model:



Approximate equivalent of the MIT system in a Tesla coil format.

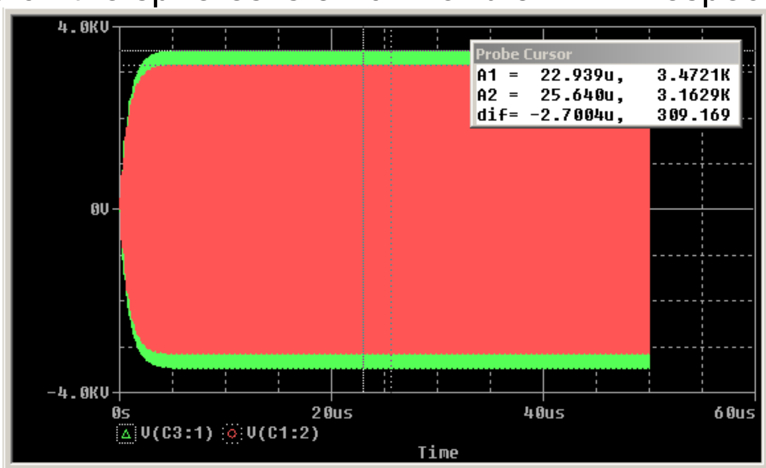
In this case, we are driving the primary Tesla coil with a typical 400 watt 10MHz 50 ohm Sine wave generator. On the receiver side, we simply have a 60 watt 12 volt (2.4 ohm) light bulb. C1 and C3 have been "tweeked" to make it all tune well at 14.0pF.

The is an important note here that the natural capacitance of the secondary coils should be 35.51pF (from MandK) and a 2 foot diameter sphere will have a capacitance of 33.87pF.

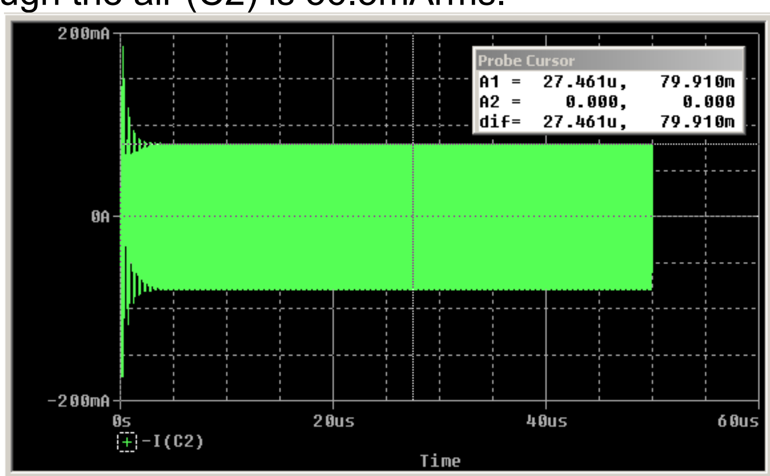
http://www.csgnetwork.com/capacspherecalc.html

It seems like the MIT system should really run at a lower frequency than 10 MHZ.

The peak voltage on the spheres is 3.16kV and 3.47 kV respectively.

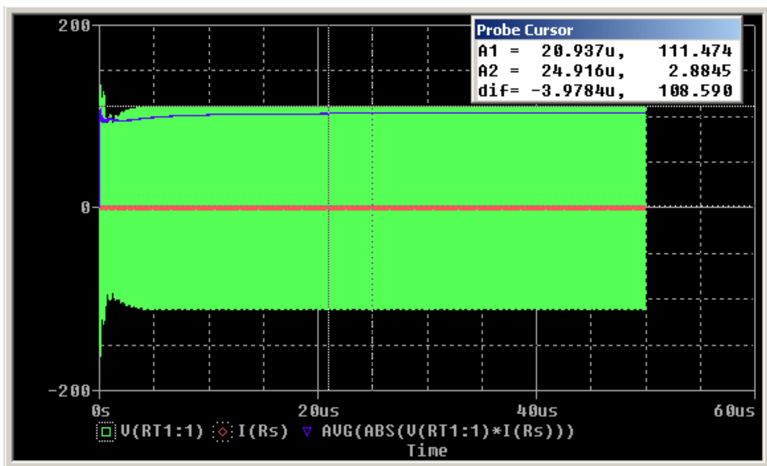


The current through the air (C2) is 56.5mArms.



The voltage and current in the 2.4 ohm load resistor are 7.95Vrms and 3.31Arms.

The delivered power to the load (light bulb) is 26.3 watts. The input to the transmitting coil is 78.82Vrms, 2.04Arms, at 103.8 watts real power input.



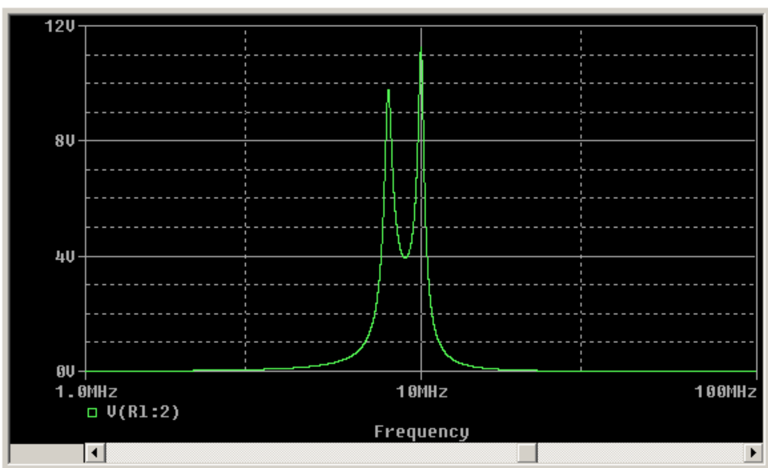
Thus the efficiency is 26.3 / 103.8 = 25.3%

terrellfoneATtwfpowerelectronics.com

UPDATE: June 13,2007

The capacitance of the secondary coils is about 1/2 what the Medhurst formula predicts since there is no ground plane in this case. In effect, there is a virtual ground plane in the center of the coil since it is acting as a 1/2 wave system rather than a 1/4 wave system. Thus, the higher frequencies the MIT system uses is very reasonable given their configuration. The pickup coil with the light bulb might do better if it were in the center of the coil rather than at an end of the coil. The fields are least in the center, so the pickup coil and bulb will have the least effect on tuning and keep they system more symmetrical if it is centered in the middle of the receiving coil.

The "Q" of the system is about 15 to 20 since both sides have a substantial load. This gives the system a tuning bandwidth of about 2.5MHz making tuning fairly easy. The fact that the bare coil has a "Q" of 1000 does not matter. We can sweep the system for input frequency vs output voltage at the bulb to see this.



System Tuning Spectrum

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