Proc. 16th Eur. Meeting on Cybernetics and Systems Research (EMCSR), Vienna, April 2 – 5, 2002, vol. 2, pp. 934-939 (= Cybernetics and Systems, R. Trappl, ed., Austrian Soc. for Cybernetics Studies).

The Scientific Work of Wilhelm Cauer and its Key Position at the Transition from Electrical Telegraph Techniques to Linear Systems Theory

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Abstract

The paper sketches a line of historical continuity ranging from telegraph pioneers and early submarine cable technology of the 19th century to some aspects of modern systems and control theory. Special emphasis is laid on the intermediary role of the scientific work of Wilhelm Cauer (1900-1945) on the synthesis of linear networks.

1 Introduction

While it is almost commonplace that modern methods of data compaction, coding and encryption in today's communications all have their roots in the techniques of the telegraph pioneers [Beauchamp, 2001], analogous facts concerning the evolution of classical network and systems theory seem to fall into oblivion. This is partly due to the dichotomy of the historical roots of modern system and control theory into analytical mechanics and into classical network theory (in the style of Bode, Cauer, Darlington, Foster, Youla, and many others) combined with a widespread unwillingness to look beyond the borders of one's own field of research. Additionally, in recent decades there has been a dramatic decline of knowledge about classical network theory that has resulted in the expulsion of the basic academic and educational aspects of network models from electrical engineering curricula and a progressive fading out of network theory as an independent scientific discipline [Rohrer, 1990].

Long before the advent of general electromagnetic theory (first formulated in 1873 by MAXWELL in his celebrated *Treatise*) the theory of electrical networks emerged as an independent discipline with original concepts and methods. These first achievements were OHM's law (1827), KIRCHHOFF's laws (1845) and the pertaining topological rules (1847), as well as the superposition principle and the so-called Thevenin-equivalent circuits credited to HELMHOLTZ (1853). Typically, a physical device such as a resistor made of metallic wire or carbon is considered a *black box* defined by a linear relation $v = Ri$ between the external electrical variables v and i . This abstraction turned the analysis of realistic physical devices into the study of sets or "systems" of well-defined idealized objects together with their mutual relations which are defined by the frequency-independent (holonomic) constraints imposed by interconnection rules. Therefore, network theory *ab initio* was set up as a purely mathematical discipline. Most notably, under the standard assumptions of linearity and time-invariance, the theory of passive networks is equivalent to the theory of dissipative mechanical systems that perform small oscillations around a stable equilibrium. Especially in his early papers on network synthesis [Cauer, 1926], [Cauer, 1931b] Cauer's mathematical starting point is this relation to classical analytical mechanics. Nevertheless, it is fair to recall that the motivation and detailed structure of the problem was firmly rooted in electrical telegraph techniques of the 19th century, especially in submarine cable transmission problems; see [Belevitch, 1962], [Darlington, 1984] and [Wunsch, 1985]. In order to keep the number of citations under control, we refer the reader to these references for more details about the historical aspects sketched in the subsequent section.

2 Before Network Synthesis

2.1 Electrical Telegraph Techniques in the 19th Century

Although network analysis has emerged as a near-mathematical discipline in the mid of the 19th century, the development of submarine telegraph cables had a very important influence on the progress of network theory, the main reasons being (i) the extremely complicated dynamical behavior of transmission cables (as compared to the ubiquitous *RLC*-circuits of modest complexity) and (ii) the boiling hot economic interest in the advancement of electrical telegraph technology — comparable only to the Internet euphoria of today. For a vivid popular account on the remarkable parallels in the history of the electrical telegraph and the revolution in communications due to the Internet, see [Standage, 1998] (though from a decidedly american viewpoint).

The first working electrical telegraph had been constructed in 1833 by GAUSS and WEBER in Göttingen using deflected needle technology, followed in 1837 by STEINHEIL's recording telegraph in Munich as well as by similar experiments in the same year by WHEATSTONE in England and MORSE in the USA. The first cable to cross the English Channel was laid in 1851, soon followed by the first deep-sea cable between Sardinia and Tunisia in 1854. When the celebrated transatlantic cable connecting Europe and America went *on-line* in 1858 misconceptions and the lack of electrical models for understanding transmission line operation posed serious practical difficulties due to an excessive spread of transmitted pulses. Telegraph engineering at that time was crude empiricism. Not even Ohm's law had found its way to many of the early experimentalists.

Various steps towards network modelling of transmission lines had been undertaken by Lord KELVIN since 1855, but his models were not well suited to explain most of the practical difficulties. After BELL's invention of the "speaking telegraph" in 1876 (preceeded about 15 years by REIS) these deficiencies became even more evident when engineers tried to transmit speech signals over long aerial or modest submarine distances. In the early 1880's there were about 150 000 kilometers of submarine telegraph cables operating by hook or by crook when HEAVISIDE proposed his celebrated analysis of transmission lines in terms of his *telegraphist's equations*

$$\begin{aligned} -\frac{\partial u}{\partial x} &= Ri + L\frac{\partial i}{\partial t}, \\ -\frac{\partial i}{\partial x} &= Gu + C\frac{\partial u}{\partial t}. \end{aligned} \qquad (1)$$

Here, R, L or G, C are the per unit length series resistance and inductance, or cross conductance and capacitance, respectively, of the well-known lumped network model for an infinitesimal piece of transmission line. Initially, Heaviside's model was not much appreciated, though in 1893 he predicted on purely theoretical grounds that a distortion-free transmission of signals is feasible by designing cables such that R, L, G, C are related in a certain manner.

2.2 First Steps Towards Network Synthesis (1900 – 1924)

At the turn of the century the situation changed almost abruptly when PUPIN confirmed by experiment the advantage of Heaviside's theoretical proposal to insert lumped inductors in the line at uniformly spaced points. Miraculously, the increased series inductance did not further deteriorate the transmission properties; rather it turned out that these *loaded lines* exhibited quite perfect transmission characteristics (up to a certain "cutoff frequency") while presenting a remarkable increase of attenuation at higher frequencies. Loaded lines, often called "Pupin-lines", became the first prototypes for low-pass filters.

The tremendous success of Heaviside's network model for understanding the complicated dynamical behaviour of transmission lines brought in new concepts such as propagation constants, attenuation and phase, impedance matching, reflections and mismatch. This renewed appreciation of Heaviside's work marked the beginning of an increasingly systematic design of communication systems by an increasing number of protagonists (BREISIG, CAMPBELL, FRANKE, HAUSRATH, KENNELLY, STRECKER, to mention just a few). Many new circuits and devices such as artificial lines, attenuators, hybrid coils, balancing networks, T- and Π-equivalent circuits for two-ports, filters, and allpass lattice sections and equalizers were designed.

At the same time complex calculus (vulgarized in electrical power engineering by STEINMETZ) became ubiquitous; rational functions and poles and zeros gradually entered into engineering papers and the notion of a multiport and its various matrix descriptions emerged. LACOUR (1903) seems to have been the first to write down the equations for a symmetric two-port in chain matrix form (that is also the solution to eqn.(1))

$$\begin{bmatrix} U_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} \cosh g & Z \sinh g \\ \frac{1}{Z}\sinh g & \cosh g \end{bmatrix} \begin{bmatrix} U_2 \\ I_2 \end{bmatrix}, \qquad (2)$$

while the general *black box* concept of a two-port was formulated by BREISIG in 1921. CAMPBELL and FOSTER established the theory of the *ideal transformer* as a new network element and gave a complete enumeration of all realizations of lossless four-ports composed of ideal transformers [Campbell, 1937, pp. 119-168].

The first breakthrough towards a systematic synthesis of filters was achieved in 1924 by FOSTER in his paper *A reactance theorem* [Foster, 1924]. The importance of this celebrated theorem rests essentially in the giving of necessary and sufficient conditions on the impedance function of a lossless one-port in order that it be realized by means of capacitors and inductors. Moreover, by use of a partial fraction decomposition Foster presents a circuit configuration (realization) that is *canonical*, in the sense that it can be used to realize any arbitrary lossless rational impedance function.

For completeness, let us remark that even Foster's 1924 paper originated from work on submarine cables. "It might be of some slight historical interest to note that the case of networks composed of resistors and capacitors was the first for which the general solution was obtained, in the very early 1920's, in connection with the design of balancing networks for submarine telephone cables. But, when written up for publication in 1924, it was presented in terms of inductors and capacitors." [Foster, 1962]

3 Cauer's Program for Network Synthesis (1926 – 1929)

It is fair to resume that many "ingredients" of a systematic synthesis theory were in the air when Cauer started his scientific career. Especially Foster's reactance theorem was a first conscious step towards a thorough mathematical description of *classes* of circuits. Cauer immediately recognized the potentialities of Foster's ideas and started a private correspondence with Foster. Already in his 1926 doctoral thesis [Cauer, 1926], he sketched a complete program for network synthesis as a solution to the *inverse* problem of circuit analysis: Given the external behavior of a linear passive one-port in terms of a driving-point impedance as a prescribed function of frequency, how does one find internally passive realizations for this 'black-box'? He then shows that the synthesis problem requires the systematic solution of three main issues concerning the *realizability*, *approximation*, and *realization* of a given impedance (voltage/current transfer function) $Z(\lambda)$ as a function of the complex frequency parameter $\lambda = \sigma + \mathrm{j}\,\omega$. At the time 'passive realization' was clearly a synonym for reciprocal circuits consisting of resistors, inductors (possibly magnetically coupled), and capacitors with the positive element values R, L and C.

Cauer had a firm education in mathematical physics, and was well acquainted with Lagrangian-style analytical mechanics, particularly in terms of the treatises of E. J. Routh and E. T. Whittaker on the dynamics of rigid bodies. With a view to investigating realizability conditions, he starts with the symmetric $n \times n$ loop matrix of a generic passive n-mesh circuit (cf. Fig. 2)

$$\boldsymbol{A} = \lambda^2 \boldsymbol{L} + \lambda \boldsymbol{R} + \boldsymbol{D}, \tag{3}$$

where $\boldsymbol{R}$, $\boldsymbol{L}$, and $\boldsymbol{D}$ are positive definite matrices of resistance, inductance, and elastance (reciprocal capacitance), respectively. The pertaining quadratic forms correspond to the rate of dissipation of energy in heat and the stored electromagnetic and electrostatic energies. Assuming the external accessible port to belong to the first mesh, the input impedance is calculated by elimination of internal variables as

$$Z(\lambda) = \frac{\det \boldsymbol{A}}{\lambda\, a_{11}}, \tag{4}$$

where a_{11} is the complement of the element A_{11} in $\det \boldsymbol{A}$. Cauer emphasizes the analogy between realizable functions and the stability theory of small oscillations in classical mechanics by comparing electrical quantities to their mechanical counterparts, the Lagrange multipliers, and the classical triple of quadratic forms (kinetic, potential and dissipated energy). Moreover, he clearly points out that on the level of the generic n-mesh circuit there are absolutely no additional realizability constraints beyond positive definiteness of the three quadratic forms when ideal transformers are admitted as circuit elements. In other words, additional constraints are exclusively imposed by the topology of the circuit.

In subsequent papers [Cauer, 1929a], [Cauer, 1929b], [Cauer, 1931b] Cauer simultaneously subjects the triple of quadratic forms to a group of real affine transformations

$$\boldsymbol{T}^{\mathrm{T}} \boldsymbol{A} \boldsymbol{T}, \quad \boldsymbol{T} = \begin{bmatrix} 1 & 0_{1,n-1} \\ T_{21} & T_{22} \end{bmatrix} \tag{5}$$

and shows that external behavior in terms of $Z(\lambda)$ in (4) is invariant! In his detailed account of Cauer's approach, N. Howitt can barely restrain his enthusiasm [Howitt, 1931]: "Considerable has been written on electrical networks and the impedance function, but it has hardly been suspected that electrical networks formed a group with the impedance function as an absolute invariant and that it was possible to proceed in a continuous manner from one network to its equivalent network by a linear transformation of the instantaneous mesh currents and charges of the network."

On the basis of this fundamentally new concept of external equivalence of passive linear networks under transformations of internal variables, Cauer was able to state the problem of linear circuit synthesis as follows [Cauer, 1941, p. 13], [Cauer, 1958, p. 49]: "The previous discussions have shown that it is less important for the electrical engineer to solve given differential equations than to search for systems of differential equations (circuits) whose solutions have a desired property. With the realization of circuits with prescribed frequency characteristics in mind and in the interest of a systematic procedure, the tasks of linear network theory are formulated as follows:

(1) *Which classes of functions of λ can be realized as frequency characteristics ?*

(2) *Which circuits are equivalent to each other, i.e. have the same frequency characteristics ?*

(3) *How are the interpolation and approximation problems (which constitute the mathematical expression of the circuit problems) solved using functions admitted under question* (1)?"

Later on, this way of studying differential equations *indirectly* through transfer functions of black-boxes and their input-output pairs became characteristic of modern linear system theory.

3.1 Two-Element Kind Networks

In his dissertation Cauer complemented Fosters reactance theorem with a more concise proof of the analytical properties of the reactance function $Z(\lambda)$ and by means of his celebrated canonical *ladder realizations* (obtained via Stieltjes' continued fraction expansions). Most notably, he adapted the results for purely reactive networks to all two-element kind networks showing an isomorphism between LC, RC and RL circuits.

Based on the observation that the poles and zeros of the pertaining $Z(\lambda)$ alternate on the real or imaginary λ-axis, he established a fundamental relationship between polynomial stability tests and realization algorithms for two-element kind circuits. In [Cauer, 1929a], he emphasizes the role of a reversal of these realization algorithms: They generate parametrizations of the pertaining subclasses of positive-real functions $Z(\lambda)$ in an algebraically trivial manner without any reference to network graphs, quadratic forms or analytic function theory.

In [Cauer, 1929b], [Cauer, 1931a], [Cauer, 1931b], [Cauer, 1934], Cauer completely solves the questions of minimal realization and equivalence for lossless reciprocal (or more generally: two-element kind) multiports with prescribed input/output behavior in two steps:

- Realization of a given n-port by partial fraction expansion of the reactance matrix $Z(\lambda)$. Due to the uniqueness of the partial fraction decomposition, this realization is canonical (minimal).
- Determination of any other externally equivalent minimal realization by equivalence transformation (5) of internal variables.

A key problem is the simultaneous principle axis transformation of two quadratic *semi*-definite forms (as opposed to the standard case where at least one form is non-degenerate). One should note that in the case of LC multiports ($\boldsymbol{R} = 0_n$) it is not difficult to write down a Kalman state space realization of the impedance matrix $Z(\lambda)$ and to show that the equivalence transformation (5) contains state space equivalence as a subgroup [Belevitch, 1968].

3.2 General Passive Multiports

As long ago as his 1926 dissertation, Cauer uses a (now) standard passivity argument to prove that boundedness of transients in an electrical circuit imposes the fundamental *necessary* realizability condition

$$\mathrm{Re}Z(\sigma + \mathrm{j}\,\omega) > 0, \quad \forall\, \sigma > 0.$$

for any impedance function $Z(\lambda)$ at 'complex frequencies' $\lambda = \sigma + \mathrm{j}\,\omega$. With regard to *rational* impedance functions, he studies the properties of elementary two-mesh circuits under the additional constraint that no ideal transformers are admitted. In an engineering exposition of his main ideas Cauer emphasizes the role of nonrational impedances: "If we do not restrict ourselves to a fixed finite number n of independent meshes but admit infinite networks, one arrives at a very simple and complete answer to the question: What is the general analytic character of a function that may be approximated with any requested precision by the driving point impedance of a finite 2-pole circuit?

The only rational functions of λ that are realizable as impedances of 2-pole networks have to be analytic in the right halfplane $\mathrm{Re}(\lambda) > 0$, have a positive real part in $\mathrm{Re}(\lambda) > 0$, and take on real values on the real axis.

These characteristics are an immediate consequence of the representation of these functions as the Poisson integral

$$Z = \lambda \Big[C + \int_0^\infty \frac{\mathrm{d}\psi(x)}{\lambda^2 + x} \Big], \qquad (6)$$

where $C \geq 0$, ψ is a monotonically increasing function, and the integral has to be taken in Stieltjes' sense. When approximating the integral by a finite sum, we get a rational function in λ that has an immediate realization as an electric circuit. [...] In this way, we obtain an *arbitrary close approximation for Z* not only in the interior of the right half plane, but also on the boundary, i.e., for purely imaginary $\lambda = \mathrm{j}\,\omega$. When a complex impedance Z is represented graphically as a function of the frequency ω, $\psi(x)$ can only be determined on the basis of the *real* part $\mathrm{Re}Z(\mathrm{j}\,\omega)$; it can be an *arbitrary, piecewise linear non-negative* function of frequency. The point is, given a preassigned real part, *the imaginary part cannot be chosen at will* if Z has to be realizable by a 2-pole circuit."

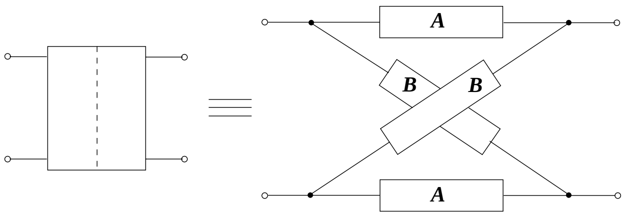

Figure 1: Equivalence of a symmetric reciprocal two-port and a lattice section [Cauer, 1927].

In essence, long before algebraic realization theory for rational impedances was producing satisfactory results of sufficient generality, Cauer obtained a complete characterization of the realizability class by means of analytic function theory (later, for this class the term *positive-real* (abbrev. PR) was coined in O. Brune's thesis [Brune, 1931]). This included (i) the (Hilbert) integral relation between real and imaginary parts of characteristic network functions as a limiting case of (6), (ii) results on their *rational* interpolation and approximation, and (iii) the relationship to function theoretical fundamentals such as Schwarz' lemma, cross ratios, non-Euclidian metrics and their contraction or invariance under mappings induced by passive or lossless circuits (cf. [Cauer, 1929a], [Cauer, 1932b]–[Cauer, 1933])

In the design of socalled image parameter filters, Cauer achieved a tremendous progress as compared to previous design methods by introducing *lattice realizations* for symmetric two-ports as shown in Fig. 1 as well as a new approximation of prescribed filter characteristics.

The lattice equivalence is a global one (i.e., independent of frequency), where the lattice impedances A and B are related to certain open and short circuit impedances Z_o and Z_s measured at the external ports of the 2-port in the following way:

$$A = Z_\mathrm{o} + \sqrt{Z_\mathrm{o}(Z_\mathrm{o} - Z_\mathrm{s})}, \quad B = Z_\mathrm{o} - \sqrt{Z_\mathrm{o}(Z_\mathrm{o} - Z_\mathrm{s})}.$$

Cauer proved that despite the irrational character of the above formulas the impedances A and B are *rational* functions of frequency and can be realized by a finite continued fraction (resulting in a passive RLC-circuit) whenever the two-port is passive. The decisive role of the lattice realization stems from possibility to replace the investigation of an important class of rational (2×2)-matrices by the treatment of two scalar rational functions. In December 1927 Cauer got the opportunity to present his lattice equivalence theorem for symmetric two-ports in a session of the Prussian Acadamy of Sciences [Cauer, 1927]. It may be instructive to illustrate the importance of this privilege by looking through the list of speakers in that year (Einstein, Hahn, Hasse, Hopf, Landau, von Neumann, Noether, Planck, Pólya, Schur ...). From a purely mathematical viewpoint, rational ma-

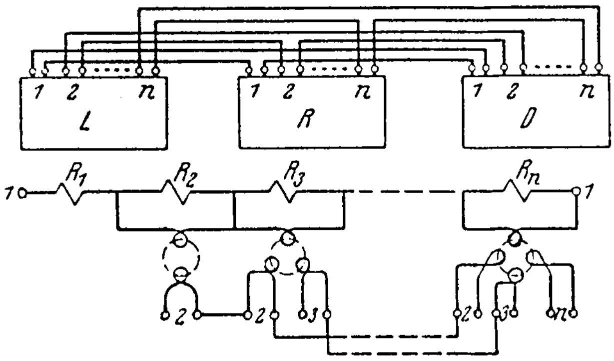

Figure 2: Canonical realization of an arbitrary finite passive multiport by three mesh-connected purely inductive, resistive and capacitive n-ports L, R and D, respectively [Cauer, 1931b]. Their internal structure is shown below for the resistive R-box, where minimality of the number of parameters is achieved by a special array of multi-winding transformers (obtained by reduction of the turns-ratio matrix to triangular form). External ports can be introduced by opening one or more of the external meshes.

trices were a rather exotic subject: Recall that the absolutely elementary notion of the (McMillan) *degree of a rational matrix* has been given a rigorous definition only 25 years later by McMillan [McMillan, 1952] based on a deceptively network-theoretic reasoning using structural results of Cauer.

On the algebraic realization side, Cauer investigated the internal structure and equivalence of multiports on the basis of (3)–(5) (cf. [Cauer, 1931b], [Cauer, 1932a], [Cauer, 1932c], and Fig. 2). In his habilitation thesis 'On a problem where three positive definite quadratic forms are related to one-dimensional complexes' [Cauer, 1931b], Cauer concentrated on the analytic properties of *RLC* multiports and the *algebraic-geometric* aspects of their canonical representation (i) in terms of rational matrices that are generated by three positive quadratic forms in n variables and (ii) in terms of their assignment to a one-dimensional cell complex of first Betti number n (in reference to Osvald Veblen's *Analysis Situs*). As Cauer points out, the main structural distinction between general *RLC* and two-element kind multiports is (i) that it is generally not possible to simultaneously diagonalize the three quadratic forms by congruence and (ii) that the occurrence of additional absolute invariants of (5) implies the non-existence of global canonical forms for the generation of all realizations. However, he showed (among other things) that the realization problem can often be split into 'smaller' ones by simultaneously transforming the quadratic forms into a common but otherwise arbitrary block-diagonal structure.

During his stay at MIT in 1930/31 Cauer suggested and supervised the doctoral thesis of Brune. By his famous continued fraction Brune provided the long-unresolved proof that the PR property is not only a necessary but also sufficient condition for a rational function to have a *physical* realization, i.e.

- in the form of a *finite* network with positive values of network elements R, L, C (or a positive definite L-matrix in case of coupled coils)
- without *ideal* transformers.

4 Conclusion

Cauer's program marked a milestone in the development of network synthesis towards a mathematical description of *classes* of circuits — and hence towards what we nowadays call mathematical systems theory. It is to Cauer's merit that he turned network and system synthesis into a truly systematic business. He provided it with the appropriate mathematical framework just as the discipline was on the rise starting from Foster's reactance theorem. In contrast to the purely utilitarian or engineering approach, Cauer emphasized the scientific aspects and provided the first precise statement of the network synthesis problem, one that has become a valid paradigm for all synthesis problems [Brockett, 1977, p. 38].

Beyond the role of positive real functions, Nevanlinna-Pick-interpolation, passivity and losslessness in modern systems and control theory (see [Dewilde and et.al. (editors), 1995]) there remains a much more important connection to classical network synthesis. Instead of solving systems of ordinary differential equations (i.e., in the tradition of classical mechanics), Foster and Cauer started solving problems indirectly by analysing *transfer functions* of black-boxes and the admissible external behavior of their input-output pairs. This way they studied classes or families of linear dynamical systems via their parametrization in terms of network models. In mathematical systems theory the first systematic studies of *families* of linear dynamical systems started in 1974 [Hermann, 1974b], [Kalman, 1974]. Although on the mathematical side there are some researchers who are aware of certain antecedents of this development in classical circuit synthesis [Brockett, 1977], [Byrnes, 1998], there has been alarmingly and amazingly little contact and understanding between these groups.

On the engineering side people are increasingly horrified by the intense structure of modern mathematical thought – it certainly cuts off access to people outside of mathematics who have not been trained to understand the style, nor have the intellectual fanaticism to immerse themselves into it (cf. Preface to [Hermann, 1974a]). It turns out, however, that a circuit engineer's intuitive way of looking at *sets* or parametrized families of multiports in terms of black-box pictures essentially amounts to the *coordinate-free* qualitative study of *global* models for differentiable manifolds [Pauli, 2000]. It seems worth it to exploit these connections more carefully and to re-examine reciprocal network models that have been studied by Cauer 75 years ago in order to understand their role in a "modern" context — for instance moduli spaces for symmetric rational transfer functions.

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