INDUCTORS AND TRANSFORMERS

We now consider the inductor. Like the capacitor, it appears in all sizes. There are significant differences, however. One is that the commercial availability of prefabricated inductors is much less than for capacitors. Engineers dealing with inductors will routinely buy coil forms and wire, and assemble their own device. With capacitors we can get by with only a vague notion of how they were designed and built. With inductors, we must do the design, which forces us to have a deeper understanding of the steps involved.

A second difference is the amount of electromagnetic theory required. With capacitors, we could escape with a mention of electric field and permittivity. Inductors require that we jump right into some challenging concepts of magnetic fields and energy, and even set up some line or volume integrals. We will refer back to a first course in electromagnetic theory as needed, and even pull out some results from more advanced courses.

1 Definitions

Consider a coil of wire as shown in Fig. 1. The resistance of the wire can be modeled as a separate lumped device so we can think of the coil as being perfectly conducting. If i is a finite dc current, the voltage v will be zero, as would be expected across a perfect conductor. If a time varying current is applied, however, there will be a related voltage observed across the coil. From the circuit theory viewpoint, the relation is given by

$$v = L\frac{di}{dt} \tag{1}$$

where L is the *inductance* of the coil in henrys (H).

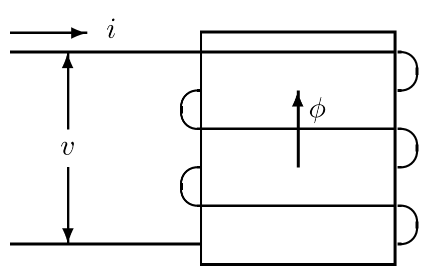

Figure 1: Simple Inductor

We will also observe a time varying voltage when we pass a magnet by the coil, even if the current is zero. (Faraday was the first to observe this, in 1831). Moving a magnet is philosophically quite different from applying a current, but it turns out that we can mathematically describe both situations with the electromagnetic equivalent of Eqn. 1.

$$v = N\frac{d\phi}{dt} \tag{2}$$

where N is the number of turns on the coil and ϕ is the magnetic flux passing through the coil. The direction of a flux ϕ that is produced by a current i is determined by the right hand rule. That is, if you curl the fingers of your right hand in the direction of the current flow, the thumb will point in the direction of the flux.

Setting Eq. 1 and Eq. 2 equal to each other and integrating to remove the differential operator yields

$$Li = N\phi \tag{3}$$

This equation has circuit quantities on the left and field quantities on the right, allowing us to move back and forth between the two ways of thinking. Solving for the inductance L gives

$$L = \frac{N\phi}{i} = \frac{\lambda}{i} \tag{4}$$

where we have introduced the *flux linkage* λ. This term better describes the case where ϕ is not constant between adjacent turns. The flux linkage can be considered as the equivalent flux which gives all the correct results when passing through a single turn coil.

The flux ϕ and the flux linkage λ are proportional to the current i. The relationship is linear if there are no ferromagnetic materials in the vicinity, which then gives us a constant value for L independent of the actual value of i. Thus for the air-cored coil, L is just a function of the geometry of the coil, much like our expression for capacitance that was calculated from area and separation of plates in the previous chapter. Unfortunately, inductance formulas tend to be much more complicated than the formula for a parallel plate capacitor.

The electric power input to the inductor is

$$p = vi = Ni\frac{d\phi}{dt} \tag{5}$$

There are no losses in our perfectly conducting coil, so whatever power flows in at one time must flow out at another time. In the sinusoidal case, power flows in for half a cycle and back out the next half cycle. The power flow results in stored magnetic field energy in the coil. The differential energy input during the differential time dt is

$$dW = p\,dt = Ni\,d\phi \tag{6}$$

where W is the stored energy in the field.

We want to relate this stored energy to the field quantities B and H, where

$$B = \mu_r \mu_o H \qquad \text{T} \tag{7}$$

B is the magnetic flux density in tesla (T) or webers/m^2 (Wb/m^2), H is the magnetic intensity in A/m, μ_r is the relative permeability (= 1 for vacuum), and μ_o is the permeability of free space in henrys per meter.

$$\mu_o \equiv 4\pi \times 10^{-7} \qquad \text{H/m} \tag{8}$$

The relative permeability is very close to unity for all materials except for the ferromagnetic materials iron, cobalt, nickel, and a number of special alloys. For these materials, μ_r may range from 10 to 10^5. The relative permeability is also a function of magnetic intensity in ferromagnetic materials, making what would be a linear problem into a nonlinear one.

The magnetic flux density B may also be expressed in gauss, where 10^4 gauss = 1 tesla. The earth's magnetic flux density varies from 0.2 to 0.6 gauss, depending on location. The 60 Hz magnetic flux density in a home or office is usually less than a few milligauss except near a source (electric heater, computer monitor, electric razor, blow dryer, etc.) where it may be a few tens of milligauss or even more. A modern well-designed 60 Hz power transformer will probably have a magnetic flux density between 1 and 2 T inside the core. It requires considerable effort and special designs to get much above 2 T. The necessary current density causes heating in the conductors, unless, of course, the conductors are cooled into the superconductor region. Fluxes as high as 8 to 16 T have been used in accelerators and energy storage systems.

Other conversion factors which might be needed are:

- 1 Oersted = $250/\pi$ = 79.6 ampere-turns/meter
- 1 Tesla = 10,000 gauss

The flux ϕ passing through an area A is the integral of the magnetic flux density B over that area.

$$\phi = \int B\, dA \qquad \text{Wb} \tag{9}$$

which becomes simply $\phi = BA$ if B is constant over the area.

We also need Ampere's circuital law

$$Ni = \oint \mathbf{H} \cdot \mathbf{d}\ell \tag{10}$$

This states that the current enclosed by any arbitrary path is given by the integral of the dot product of the vector $\mathbf{H}$ and a differential length $\mathbf{d}\ell$ along that path. If we extend our coil around into a toroid shape, H (the magnitude of $\mathbf{H}$) will be essentially constant inside the toroid and Ampere's circuital law becomes

$$Ni = H\ell \tag{11}$$

where ℓ is the length of a circle in the toroid. The energy stored in the magnetic field is now

$$dW = Ni\,d\phi = \frac{H\ell}{i} iA\,dB = (\mathrm{Vol})\,H\,dB \tag{12}$$

where Vol $= A\ell$ is the volume where the magnetic energy is stored. The total energy can be found by integration.

$$W = (\mathrm{Vol})\int_0^B H\,dB \qquad \text{Joules} \tag{13}$$

If the permeability is constant (the magnetic circuit is linear) the integral can be quickly evaluated.

$$W = \frac{(\mathrm{Vol})}{\mu_r\mu_o}\frac{B^2}{2} = (\mathrm{Vol})\,\mu_r\mu_o\frac{H^2}{2} \qquad \mathrm{J} \tag{14}$$

This equation has much important information in it. Suppose that we have a magnetic circuit that is entirely ferromagnetic. H is determined by the current and is independent of the permeability. $B = \mu H$ is large and the total energy stored is large. Suppose now that we cut a small air gap across the magnetic circuit. The flux ϕ drops substantially because of the increased reluctance of the magnetic circuit. B will have about the same (smaller) value in both the iron and the air gap, so $H = B/\mu$ will be much larger in the air gap because of the lower permeability. The total integral of $\mathbf{H}\cdot\mathbf{d}\ell$ stays the same but a large fraction of the integral comes from the air gap portion. So the total energy stored decreases as the air gap length increases, and the fraction of the total energy stored in the air gap increases dramatically.

Although not as instructive, the total energy can also be expressed in circuit quantities as

$$W = \frac{1}{2}Li^2 \qquad J \tag{15}$$

We can also use the result from Ampere's circuital law to determine the flux ϕ. In the simple case of uniform flux density B and no air gap, this becomes

$$\phi = BA = \mu_r\mu_o HA = \frac{\mu_r\mu_o NiA}{\ell} \tag{16}$$

2 Ferromagnetic Losses

Two things happen when a time varying magnetic field exists inside a ferromagnetic material. Magnetic domains rotate in the material to align with the magnetic field and the Faraday voltage induced inside the material produces what are called eddy currents. The rotation of domains each cycle produces a frictional type loss called the hysteresis loss P_h. Experimentally, we find that

$$P_h = K_h f B_{max}^z \tag{17}$$

where K_h is an empirical property of the material, f is the frequency, B_{max} is the maximum flux density, and z is an empirically determined value, usually between 1.6 and 2.0 for power frequency transformer steels.

The hysteresis loop of the material is obtained by plotting the magnetic flux density B against the magnetic intensity H as shown in Fig. 2. The area inside the hysteresis loop is the energy dissipated each cycle. Some ferromagnetic materials have very thin hysteresis loops, resulting in low losses, while others have relatively fat loops and correspondingly high losses.

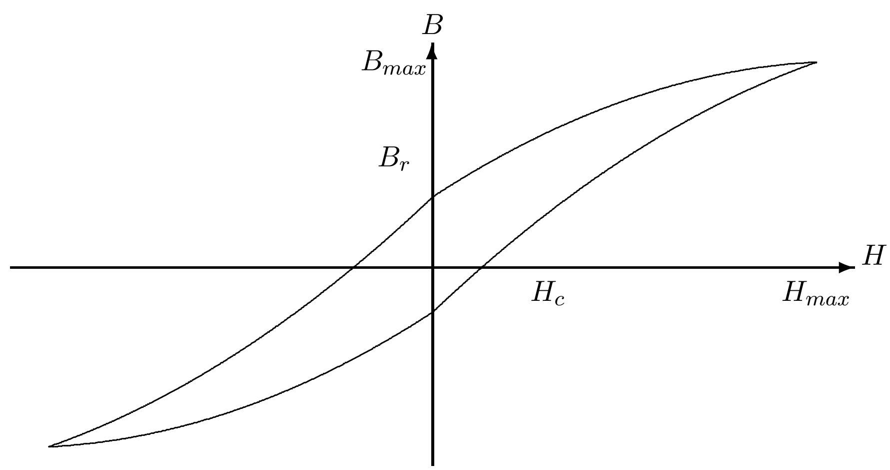

Figure 2: Hysteresis Curve

As the driving magnetic field increases, the resulting flux density increases at a slower rate as more domains become aligned with the magnetic field. This phenomenon is called *saturation*. If linearity is desired, then the transformer should be operated at low flux levels where the hysteresis loop is nearly linear. For power transfer, however, it is most cost effective to operate the device into its saturated region. The exact amount is a matter of engineering judgment.

We might define at least two definitions for permeability, from which we can get some guidelines for saturation. These are the dc and ac permeabilities

$$\mu_{dc} = \frac{B}{H} \tag{18}$$

$$\mu_{ac} = \frac{\Delta B}{\Delta H} \tag{19}$$

The two permeabilities are identical for very low drive levels that are symmetric about zero. If a dc bias exists, the ac permeability will always be smaller than μ_{dc}. The ac permeability is the main parameter of interest to filter choke designers. At least some engineers consider a material to be saturated when the dc permeability has dropped to half its initial value, or when the ac permeability has dropped to one-eighth of its initial value [12, page 26,28].

A magnetic circuit with eddy currents is shown in Fig. 3. The current is inversely proportional to the resistance seen by the induced Faraday voltage. In a large piece of steel the resistance can be very low, even though the resistivity of steel is not very low compared with a good conductor like copper. For this reason, magnetic circuits at power frequencies are usually made of thin sheets of steel, called laminations, which are on the order of 0.5 mm thick.

The equation for eddy current loss has the form

$$P_e = K_e f^2 B_{max}^2 \tag{20}$$

The experimentally determined constant K_e depends on the resistivity and the dimensions of the material. A detailed analysis shows that K_e is proportional to the square of the lamination thickness, so it is important to keep the laminations as thin as possible. Eddy current losses can be kept acceptably low at 60 Hz with little difficulty, but become excessive at a few kHz, even with very thin laminations. Therefore, inductors or transformers built for operation above 1 kHz are rarely made of laminated material. Instead, they are made of even smaller pieces of ferromagnetic material, typically powdered iron or ferrites. Powdered iron suffers from low permeability and low resistivity compared with ferrites, so we shall concentrate on the latter.

3 Ferrites

Ferrites were developed during and after World War II. The chemical formula for ferrites is $\mathrm{Z\,Fe_2\,O_4}$, where Z stands for any of the divalent ions: zinc, copper, nickel, iron, cobalt, manganese, or magnesium, or a mixture of these ions. The bulk resistivities are in the range of 10^2 to 10^9 ohm-cm, compared with 10^{-5} ohm-cm for powdered iron. This very high resistivity reduces the eddy current losses so that ferrites can be used for frequencies up to 20 MHz or even more.

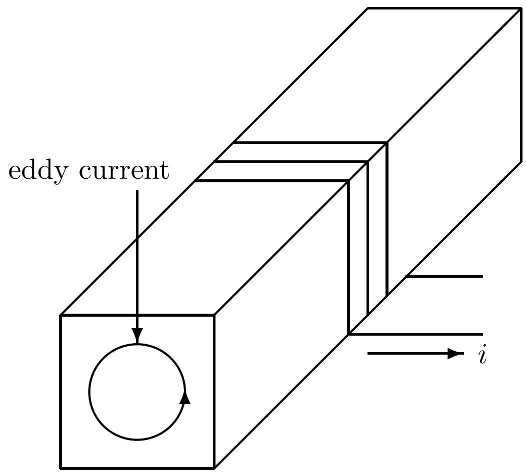

Figure 3: Eddy Currents in Magnetic Circuit

Ferrites are basically a form of ceramic, made by mixing fine powders of appropriate oxides, compressing the mixture, and firing it in carefully controlled atmospheres at temperatures of about 1100^{o}C to 1200^{o}C. The most common ferrites are mixtures of two ferrite powders, either manganese-zinc or nickel-zinc. Magnetic properties can be varied over a significant range by changing the ratio of the two divalent ions and by changing the processing conditions. Each material is given a unique code number. Ferroxcube, for example, assigns a "3" as the first digit of its MnZn materials (3C85, 3B7, 3D3, etc.) and a "4" as the first digit of NiZn materials (4C4, 4A, 4A6, etc.)

3.1 Ferrite Temperature Limits

The Curie temperature (the temperature at which the ferrite becomes nonmagnetic) can be relatively low. The Ferroxcube 3E5 ferrite may have a Curie temperature as low as 120^{o}C. Other Ferroxcube materials have Curie temperatures up to 300^{o}C. If there is any possibility of the ferrite device operating at a temperature above 120^{o}C due to internal losses, a ferrite material with an adequate Curie temperature must be selected.

The copper wire used in winding inductors or transformers remains mechanically stable at temperatures far above the ferrite Curie temperature, so the wire itself is not of concern. However, the insulation on the wire may fail at relatively low temperatures. Polyvinyl chloride (PVC) insulated wire typically has a maximum temperature rating between 80^{o}C and 105^{o}C, for example. Magnet wire has a somewhat higher rated temperature, such as the Belden polythermaleze coating rated at 180^{o}C. Belden also makes Teflon coated wires rated at 200^{o}C and 260^{o}C.

Heat is dissipated from the surface of the inductor or transformer by a combination of radiation and convection. Heat radiated depends on the device surroundings, while convection depends on air flow over the device, so it is very difficult to accurately predict temperature rise in most installations. In any case, both radiation and convection will be directly proportional to the total exposed surface area of the core and windings. We can therefore describe the independent variable as total watts dissipated in core and copper per unit area of the device. The dependent variable, temperature rise, is directly proportional to the dissipation in W/cm^2. One manufacturer (Magnetics) has calculated a temperature rise of 10^{o}C for a surface dissipation of 0.01 W/cm^2 and a rise of 100^{o}C for a surface dissipation of 0.1 W/cm^2, given some reasonable assumptions. Each increment of 0.01 W/cm^2 results in a temperature rise of 10^{o}C.

Example.

An inductor has a total heat dissipation of 0.06 W/cm^2 and is in an ambient temperature of 50^{o}C. What is a reasonable estimate of inductor temperature?

Based on the Magnetics guideline, 0.06 W/cm^2 should yield a 60^{o}C temperature rise above the ambient, so the inductor temperature will be $60 + 50 = 110^o$C. PVC insulated wire should not be used in this situation.

4 Mutual Inductance

Consider two inductively coupled coils as shown in Fig. 4. The current i_1 produces a flux ϕ_{11} that links with i_1. Part of ϕ_{11} is lost as *leakage flux* $\phi_{1\ell}$ and part of it, flux ϕ_{21} links both currents i_1 and i_2. Current i_2 likewise produces a flux ϕ_{22} with part of it, flux ϕ_{12}, that links both currents. The relationship among these fluxes is

$$\phi_{11} = \phi_{21} + \phi_{1\ell} \tag{21}$$

$$\phi_{22} = \phi_{12} + \phi_{2\ell} \tag{22}$$

The self-inductance of circuit 1 is

$$L_{11} = \frac{N_1 \phi_{11}}{i_1} \tag{23}$$

and similarly for L_{22}. The mutual inductance of circuit 1 with respect to circuit 2 is based on the flux in circuit 1 that is produced by the current in circuit 2.

$$L_{12} = \frac{N_1 \phi_{12}}{i_2} \tag{24}$$

and similarly for L_{21}.

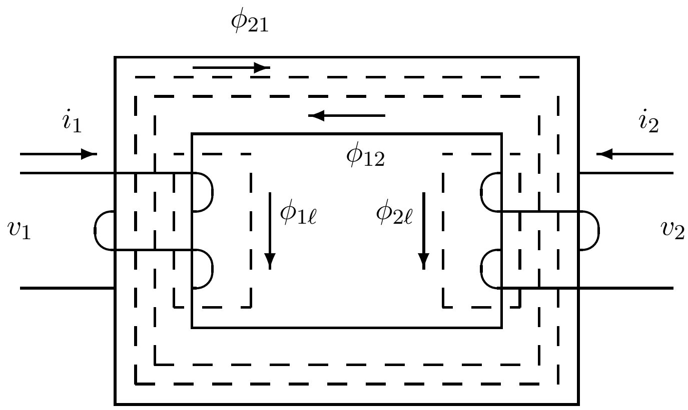

Figure 4: Mutual Inductance

It can be shown that $L_{12} = L_{21}$ in a homogeneous medium of constant permeability. To emphasize this fact we define a new symbol M for the mutual inductance.

$$M = L_{12} = L_{21} \tag{25}$$

The maximum value for M is $\sqrt{L_{11}L_{22}}$. We define the *coefficient of coupling* k as

$$k = \frac{M}{\sqrt{L_{11}L_{22}}} \tag{26}$$

The coefficient of coupling can reach values as high as 0.998 in iron-core transformers. It is difficult to make k much above 0.5 in air-core transformers.

The voltage v_2 produced by the primary current i_1 is given by

$$v_2 = M\frac{di_1}{dt} \tag{27}$$

If the two inductors of Fig. 4 are connected in series, the total inductance is

$$L = L_{11} + L_{22} \pm 2M \tag{28}$$

where the plus or minus is determined by whether the mutual flux tends to reinforce or cancel the fluxes of the individual coils. This is a convenient method to measure the mutual inductance. Just measure the series inductance twice, once with one coil reversed, subtract one result from the other, and solve the resulting expression for M.

5 Inductance Formulas

Let us now examine the inductance formulas for some simple geometries. First we will look at the inductance of a nonmagnetic coaxial transmission line. The radius of the inner conductor is a, and the inside radius of the outer conductor is b. From Ampere's circuital law, it is easy to show that, for $a < r < b$,

$$H = \frac{I}{2\pi r} \qquad \text{A/m} \tag{29}$$

and therefore

$$B = \mu_o H = \frac{\mu_o I}{2\pi r} \qquad \text{T} \tag{30}$$

We cannot use $\phi = BA$ since B varies from inner to outer conductor. Instead, we integrate to find the flux crossing any radial plane extending from $r = a$ to $r = b$ and from, say, $z = 0$ to $z = \ell$.

$$\phi = \int B\, dS = \int_0^{\ell} \int_a^b \frac{\mu_o I}{2\pi r}\, dr\, dz = \frac{\mu_o I \ell}{2\pi} \ln \frac{b}{a} \qquad \text{Wb} \tag{31}$$

The flux links the current once, so $N = 1$. From Eqn. 4 the inductance for this length ℓ is

$$L = \frac{\phi}{I} = \frac{\mu_o \ell}{2\pi} \ln \frac{b}{a} \qquad \text{H} \tag{32}$$

Suppose now that we try to use this expression to find the inductance of a segment of isolated straight conductor. As the radius of the outer conductor $b \longrightarrow \infty$, the corresponding inductance also becomes infinite. What this result tells us is that we never actually have a isolated straight conductor carrying a current without some return path. We must always consider the return path for current if we expect to get inductance values that have any relationship to reality.

In the situation of a toroidal coil of N turns and a current I, as shown in Fig. 5, the magnetic flux density is

$$B = \frac{\mu_r \mu_o N I}{2\pi r} \qquad \text{T} \tag{33}$$

For a toroid cross section that is rectangular, as shown in Fig. 6, the integration is straightforward.

$$\phi = \int_{r=a}^{b} \int_{z=0}^{T} \frac{\mu_r \mu_o N I}{2\pi r} dr\, dz = \frac{\mu_r \mu_o N I T}{2\pi} \ln \frac{b}{a} \tag{34}$$

We then multiply the flux by N to get the total flux linkages, and divide by I to get the inductance. For the rectangular cross section case, this is

$$L = \frac{\mu_r \mu_o N^2 T}{2\pi} \ln \frac{b}{a} \qquad \text{H} \tag{35}$$

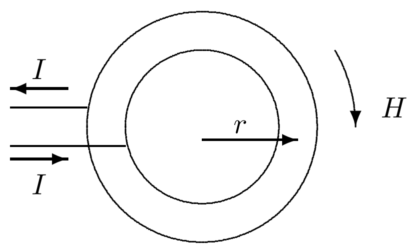

Figure 5: Toroidal Coil

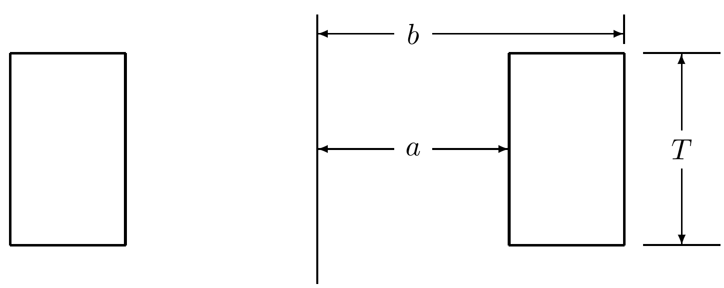

Figure 6: Toroidal Coil Cross Section

5.1 Tesla Coil Inductance (Wheeler)

Many times we just want to make a quick estimate of the inductance of some simple structure without making extensive calculations. Many approximate formulas have been developed in the days before hand calculators and computers, of which one will be given here. Watch out for the fact that the above formulas are given in the standard SI units, but the following formula is in the English system.

The low-frequency inductance of a single-layer solenoid is approximately [17, p. 55].

$$L_w = \frac{r^2 N^2}{9r + 10\ell} \;\; \mu\text{H} \tag{36}$$

where r is the radius of the coil and ℓ is its length in inches. This formula is accurate to within one percent for $\ell > 0.8r$, that is, if the coil is not too short. It is known in the Tesla coil community as the Wheeler formula. The structure of a single-layer solenoid is almost universally used for Tesla coils, so this formula is very important. In normal conditions (no other coils and no significant amounts of ferromagnetic materials nearby) it is quite adequate for calculating resonant frequency.

Example

What is the approximate inductance of an air-cored solenoid with $r = 8$ inches, $\ell = 30$ inches, and $N = 175$ turns?

$$L = \frac{(8)^2(175)^2}{9(8) + 10(30)} = \frac{1960000}{372} = 5270 \ \mu\text{H}$$

If one needs the inductance for other geometries, Terman [17] has a number of expressions. A somewhat more recent paper by Fawzi and Burke [3] gives formulas for calculating the self and mutual inductances of circular coils in a form suitable for computer calculation.

6 High Frequency Transformers

Conventional low frequency transformers consist of coils of wire wound around steel laminations. As mentioned earlier, the losses, especially the eddy current losses, become excessive at frequencies above a few kHz with this technology. Transformers built for operation at frequencies above a kHz or so are built around ferrite cores or air cores. Ferrite cores yield very compact, efficient transformers. Saturation limits operation to moderate power levels, however. If extremely large currents or powers are involved, then the air core transformer may be the logical choice. We shall discuss both types.

The symbolic construction of a transformer is shown in Fig. 7. A voltage v_1 produces a current i_1 which in turn produces a flux ϕ_1. Part of ϕ_1 links the secondary winding and produces a voltage v_2 by Faraday's Law. If some load is connected, a current i_2 will flow. This current produces a flux which opposes the original ϕ_1 according to Lenz's Law. This reduces the voltage induced in the primary so that more primary current will flow for a given source voltage. A complete description of transformer action in terms of field quantities has been developed only rather recently [4, 6, 9, 10, 16].

7 The Ideal Transformer

It will be convenient to describe the actual transformer in terms of an *ideal* transformer. This is a transformer with no copper losses, no hysteresis or eddy current losses, and perfect magnetic coupling between primary and secondary. For such a device, the relationships between input and output voltages and currents are

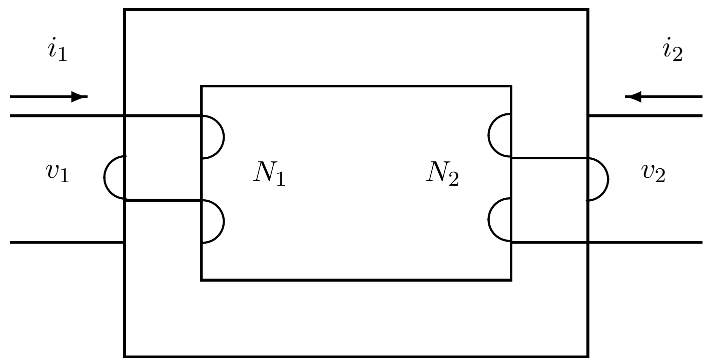

Figure 7: A Two Winding Transformer

$$\frac{v_1}{v_2} = \frac{N_1}{N_2} \tag{37}$$

$$\frac{i_1}{i_2} = \frac{N_2}{N_1} \tag{38}$$

The relationship between the input and output apparent power is

$$v_1 i_1 = v_2 \frac{N_1}{N_2} i_2 \frac{N_2}{N_1} = v_2 i_2 \tag{39}$$

The input impedance is

$$Z_1 = \frac{v_1}{i_1} = \frac{v_2 N_1/N_2}{i_2 N_2/N_1} = Z_2 \frac{N_1^2}{N_2^2} \tag{40}$$

The ideal transformer thus changes the level of voltage, current, and impedance between primary and secondary.

8 The Actual Transformer

A complete circuit model of the actual transformer is shown in Fig. 8.

In Fig. 8, R_1 and R_2 are the resistances of the primary and secondary windings, L_1 and L_2 are the leakage inductances, R_m is an equivalent resistance representing the hysteresis and eddy current losses, and L_m is the magnetizing inductance. The circuit is usually simplified by eliminating the ideal transformer and replacing all the impedances, voltages, and currents

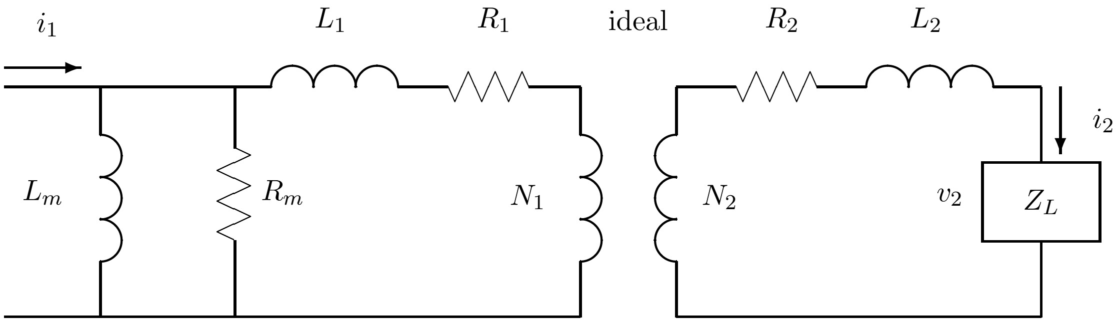

Figure 8: Transformer Model

on the secondary side with their equivalent values as seen by the primary. If we define the turns ratio a as

$$a = \frac{N_1}{N_2} \tag{41}$$

the simplified circuit is as shown in Fig. 9.

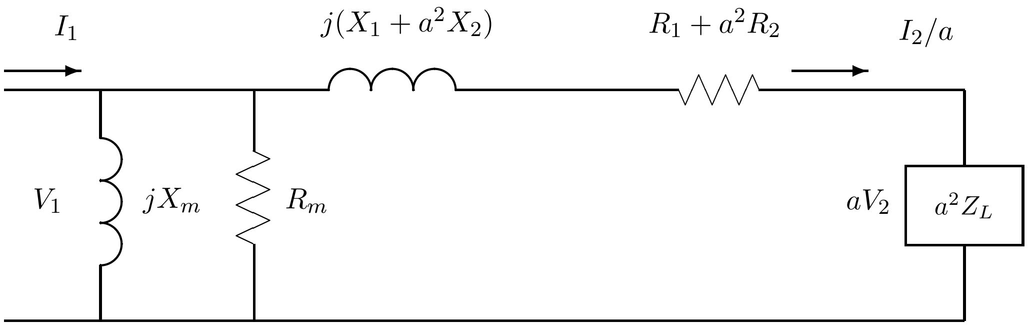

Figure 9: Actual Transformer Referred to Primary

We have also shifted to the phasor notation in Fig. 9, replacing inductances by their equivalent reactances ($X_1 = \omega L_1$, etc.), and the instantaneous voltages and currents by the phasor quantities. The notation can be shortened even more by defining an equivalent resistance and reactance

$$R_{eq} = R_1 + a^2R_2 \tag{42}$$

$$X_{eq} = X_1 + a^2 X_2 \tag{43}$$

The load current is given by

$$\frac{I_2}{a} = \frac{V_1}{R_{eq} + jX_{eq} + a^2 Z_L} \tag{44}$$

The copper losses are given by

$$P_{copper} = (I_2/a)^2 R_{eq} \tag{45}$$

and the core losses are given by

$$P_{core} = \frac{V_1^2}{R_m} \tag{46}$$

Other conducting materials can be used, such as aluminum, but it is tradition to refer to these series losses as copper losses regardless of the conducting material. It is good practice to keep the copper losses and core losses within a factor of two of each other, at least on large power transformers. The two losses tend to work against each other in a design. The core losses are reduced by reducing the maximum magnetic flux density in the core, which requires either a larger core cross-sectional area or more turns on each winding. Either approach requires more wire, which increases the copper loss. There may be instances where the core losses in a ferrite core used at high frequencies are much higher than the copper losses in reasonably sized wire. In such cases, one should go ahead with proper sizes rather than try to reduce the wire size and increase the copper losses to attempt to maintain some arbitrary parity.

The air core transformer has no core losses, of course. R_m can be removed from Fig. 9 in such cases.

9 Transformer Design

Now we are ready to design a simple transformer. We want to select wire sizes, core size, core material, and number of turns on primary and secondary so that the transformer will meet the requirements without overheating, but without being so large that it is more expensive than necessary.

The most important rating is the required voltage of operation. This determines core size and material, and the number of turns. The wire size is then selected to handle the transformer current rating without excessive copper losses. As we have seen before, the voltage is related to the flux by Faraday's Law.

$$v_2 = N_2 \frac{d\phi}{dt} \tag{47}$$

To a good approximation, the input and output voltages are sinusoidal. That is,

$$v_2 = \sqrt{2} V_2 \cos \omega t \tag{48}$$

where V_2 is the rms output voltage.

The flux is then found by integration.

$$\phi = \frac{1}{N_2} \int v_2 dt = \frac{\sqrt{2} V_2}{\omega N_2} \sin \omega t \tag{49}$$

The flux density B is given by $B = \phi/A$, where A is the cross- sectional area of the core. The maximum flux density, B_{max}, is found when $\sin \omega t = 1$.

$$B_{max} = \frac{\sqrt{2} V_2}{\omega N_2 A} \tag{50}$$

B_{max} is determined either by published data for a particular type of magnetic material, or by measurement. It is typically in the range of 1 T for low frequency laminated transformer steel, and in the range of 0.1 to 0.3 T for ferrite cores. The most efficient use of the magnetic material occurs when B_{max} is slightly above the knee of the magnetization curve. As the material saturates, the permeability $\mu = B/H$ starts to decrease. The inductance is directly proportional to permeability, so the inductance starts to decrease also. But the inductance is defined as $L = N\phi/i$. The flux ϕ is proportional to the sinusoidal voltage so it does not saturate. Therefore, as the permeability and the inductance decrease, the current i must increase. The peak of the magnetizing current increases rapidly above the knee of the magnetization curve, and can exceed the peak of the rated current if the transformer voltage is increased too far. The magnetizing current becomes very nonsinusoidal, with a high harmonic content, at higher voltages. As a rough guideline, the peak of the magnetizing current should not exceed perhaps 10 % of the peak of the rated current. That is, if the rated current were 5 A rms, with a peak of $\sqrt{2}(5) = 7.07$ A, then the peak of the magnetizing current should not exceed about 0.7 A.

Once we know B_{max}, the minimum number of turns can be determined from the above equation.

$$N_{2,min} = \frac{\sqrt{2} V_2}{\omega B_{max} A} \tag{51}$$

Example.

A Ferroxcube 204XT250-3F3 ferrite core is to be used for a transformer at 50 kHz. The rms input and output voltages are to be 10 V. Determine the proper number of turns on each winding.

We first examine a published hysteresis curve for this material, which indicates that saturation is acceptable up to about 0.3 T, at least for low frequency operation. We then check a chart of core loss versus flux density which shows a recommended operating range of 100 to 300 mW/cm^2 for this material. If the frequency is above about 25 kHz, then B_{max} must be reduced to maintain the total heating in this range. At 50 kHz, $B_{max} = 0.2$ T causes a heating of slightly under 200 mW/cm^2, which is deemed acceptable. From the published mechanical data, the area A is 0.148 cm^2. The minimum number of turns is then

$$N_{2,min} = \frac{\sqrt{2}(10)}{2\pi(50 \times 10^3)(0.2)(0.148 \times 10^{-4})} = 15.2 \ \text{turns}$$

We would normally round up to the next higher integer, 16 turns.

Suppose we were interested in using the same ferrite core for the same 10 V transformer at 60 Hz. Assuming $B_{max} = 0.3$ T, the minimum number of turns is

$$N_{2,min} = \frac{\sqrt{2}(10)}{2\pi(60)(0.3)(0.148 \times 10^{-4})} = 8450 \ \text{turns}$$

Any attempt to wind this many turns through a toroid opening of only 0.312 inches inside diameter would be frustrating at best. This points out the fact that low frequency transformers must be relatively large.

The power rating of a transformer can be determined from Ampere's circuital law and Faraday's law, which state, for the sinusoidal case,

$$H\ell = Ni \tag{52}$$

$$v = N\frac{d\phi}{dt} = N\phi_{max}\omega\cos\omega t \tag{53}$$

Converting these equations to rms values yields the apparent power

$$S = VI = \omega NBA(\frac{H\ell}{N}) = \omega BHA\ell = \omega BH(Vol) \tag{54}$$

where B and H are rms values and Vol is the volume of the magnetic material. We see that the apparent power is directly proportional to the frequency and to the volume of the transformer. This helps explain why aircraft use 400 Hz rather than 60 Hz. The transformer volume and mass are reduced by the same ratio, thus increasing the aircraft payload.

Example.

The Ferroxcube core of the previous example has a volume of 0.462 cm^3 and a relative permeability of 1800. What is the power rating at 50 kHz and a $B_{max} = 0.2$ T?

$$VI = \omega \frac{B_{max}}{\sqrt{2}} \left(\frac{B_{max}}{\sqrt{2}\mu_r \mu_o} \right) (Vol) = \frac{2\pi(50,000)(0.2)^2(0.462 \times 10^{-6})}{(2)(1800)(4\pi \times 10^{-7})} = 1.283 \quad \text{W}$$

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