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RAPID COMMUNICATION

Two-dimensional simulation of streamers using the FE-FCT algorithm

G E Georghiou†, R Morrow‡ and A C Metaxas†

† Electricity Utilization Group (EUG), Department of Engineering, University of Cambridge, Trumpington Street, Cambridge CB2 1PZ, UK
‡ CSIRO, Division of Telecommunications and Industrial Physics, PO Box 218, Lindfield NSW 2070, Australia

E-mail: `geg1000@eng.cam.ac.uk`, `richard.morrow@tip.csiro.au` and `acm@eng.cam.ac.uk`

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Abstract. The improved finite-element flux-corrected transport method (FE-FCT), developed by the authors, has been applied in its full two-dimensional form to the numerical solution of streamer development and propagation in air at atmospheric pressure. The numerical algorithm used employs the FE-FCT method for the solution of transport equations of charged species under the action of space-charge electric field, with the field obtained from the solution of Poisson's equation in cylindrical co-ordinates.

Results of two-dimensional simulations of cylindrically symmetric streamers in air between parallel plate electrodes are presented. The breakdown voltages predicted by the model for different gap lengths are in agreement with the experimental measurements, which demonstrates the validity of the model. Furthermore, the avalanche-to-streamer transition—as described in Raether H 1964 *Electron avalanches and breakdown in gases* (London: Butterworths)—and the formation of anode and cathode directed streamers are numerically observed.

1. Introduction

A considerable amount of theoretical, numerical and experimental effort has been devoted to the understanding of the development of an electron avalanche, its transition into streamers and the propagation of streamer fronts [2, 6, 10, 11]. Although kinetic models give a clear physical picture of the avalanche-to-streamer transition and its subsequent propagation, the vast majority of numerical simulations of such phenomena have used hydrodynamic diffusion-drift approximation models to describe the evolution of charge densities. This is mainly due to the fact that kinetic simulations are time consuming.

Within the framework of a hydrodynamic diffusion-drift model, the simplest set of equations containing the basic physics necessary for streamer formation and propagation are the continuity equations for electrons, positive ions and negative ions (to account for the development of the space-charge), coupled with Poisson's equation (to account for the modification of the electric field due to space-charge).

In numerical studies, difficulties arise with the solution of the continuity equations, due to the very steep shock-like gradients that appear in such calculations, and, as a result, a very accurate numerical technique is required to capture their development. Consequently, most of the existing algorithms for the solution of gas discharge problems are based on the FCT method, as this achieves the capture of steep gradients without introducing spurious oscillation or artificial diffusion.

Until recently, the characterization of gas discharge phenomena has been carried out through the finite-difference (FD) method in conjunction with the FCT (either in one dimension for the continuity equation and two dimensions for the Poisson's solution [8] or, more recently, using fully two-dimensional models [2, 6, 10, 11]). The structured nature of the grids associated with the FD method, results in a large number of unknowns and, subsequently, long calculations when applied to such complex models. There was a need, therefore, to develop new numerical methods to overcome these shortcomings. It is with such a background that the FE-FCT method was developed further and extended to gas discharge problems, by the authors [3, 4].

The use of unstructured grids, associated with finite elements, reduces significantly the number of unknowns and hence computing time, for a given problem, through the use of fine resolution where the steep gradients occur. The modelling of complex geometries is then made possible, maintaining at the same time the ability to capture these steep gradients (through the FCT part).

The FE-FCT method in one dimension has been previously coupled with a finite-element solution of Poisson's equation, in order to characterize streamer formation in

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short gaps [3]. The results obtained were compared to FD-FCT results with good agreement. However, there are limitations in the applicability of the results, due to the errors introduced into the calculation of the electric field by the one-dimensional approximation of the charges.

In this communication, the results of the full two-dimensional algorithm (employing the FE-FCT described in [4]) applied to problems involving avalanche-to-streamer transition and streamer propagation, in parallel plate electrodes in air at atmospheric pressure are presented. The algorithm is firstly validated and tested under different conditions by comparing the experimental breakdown voltages for different gap lengths with the predicted values. The avalanche-to-streamer transition and streamer propagation (both cathode and anode directed streamers) are numerically demonstrated.

2. Simulation model

The basic dynamical equations for streamer formation and propagation are the continuity equations for electrons and ions. The electric field is solved using Poisson's equation. These equations take the form:

$$\frac{\partial N_\mathrm{e}}{\partial t} = S + N_\mathrm{e}\alpha|\boldsymbol{W}_\mathrm{e}| - N_\mathrm{e}\eta|\boldsymbol{W}_\mathrm{e}| - N_\mathrm{e}N_\mathrm{p}\beta - \nabla\cdot(N_\mathrm{e}\boldsymbol{W}_\mathrm{e}) + \nabla\cdot(\boldsymbol{D}\nabla N_\mathrm{e}) \tag{1}$$

$$\frac{\partial N_\mathrm{p}}{\partial t} = S + N_\mathrm{e}\alpha|\boldsymbol{W}_\mathrm{e}| - N_\mathrm{e}N_\mathrm{p}\beta - N_\mathrm{n}N_\mathrm{p}\beta - \nabla\cdot(N_\mathrm{p}\boldsymbol{W}_\mathrm{p}) \tag{2}$$

$$\frac{\partial N_\mathrm{n}}{\partial t} = N_\mathrm{e}\eta|\boldsymbol{W}_\mathrm{e}| - N_\mathrm{n}N_\mathrm{p}\beta - \nabla\cdot(N_\mathrm{n}\boldsymbol{W}_\mathrm{n}) \tag{3}$$

$$\nabla\cdot(\epsilon_\mathrm{r}\nabla V) + \frac{e}{\epsilon_0}(N_\mathrm{p} - N_\mathrm{n} - N_\mathrm{e}) = 0 \tag{4}$$

where t is the time, N_e, N_p and N_n are the electron, positive ion and negative ion densities respectively, and $\boldsymbol{W}_\mathrm{e}$, $\boldsymbol{W}_\mathrm{p}$ and $\boldsymbol{W}_\mathrm{n}$ their drift velocities. The symbols α, η, β and $\boldsymbol{D}$ denote the ionization, attachment, recombination and electron diffusion coefficients, respectively. The source term, S, describes the effect of any particle source and sink mechanisms, such as photoionization. We have not included photoionization in the calculations presented here and we have set S to 0. Photoemission at the cathode is, however, included as a boundary condition [5]. In Poisson's equation (4), ϵ_0 is the dielectric constant of free space, ϵ_r the relative permittivity, e the electron charge and V the electric potential. The electric field E is computed using:

$$E = -\nabla V. \tag{5}$$

The transport properties of the gas (such as α and $\boldsymbol{W}_\mathrm{e}$) are unique and empirically determined functions of the ratio E/N, where E is the local electric field and N is the neutral gas number density. The expressions fitted to the physical data are given elsewhere [8].

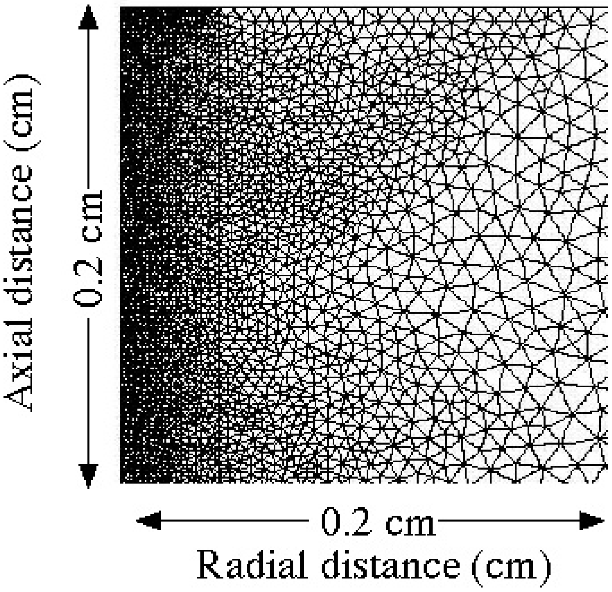

Figure 1. A typical unstructured grid used for the simulations.

3. Numerical Method

The algorithm uses the standard Taylor–Galerkin method for the solution of Poisson's equation, together with a new improved finite element-flux corrected transport (FE-FCT) algorithm for the continuity equations. The two-step Lax–Wendroff method is used as the high order scheme in the FE-FCT, and mass diffusion is added to transform it to low order. The diffusion coefficient inherent in the upwind differencing scheme is employed, as it is the optimal for the performance of the FCT method. Further details can be found in [4].

There are two distinct advantages offered by this algorithm. Firstly, the significant reduction of unknowns and the accurate modelling of boundaries are achieved through the use of unstructured grids, typically shown in figure 1. In the region around the axis of symmetry, very fine resolution is used, in order to resolve the steep gradients during the streamer propagation, but away from it very coarse mesh is used in order to make the calculation efficient, as the charge in this region is low. Approximately, 4300 unknowns are used which is about one order of magnitude lower than the number of unknowns required for the finite difference method for the same problem.

Secondly, with the finite-element method, the solution of Poisson's equation is reduced to a single matrix vector multiplication, in which the matrix depends only on the mesh used and the applied voltage and the vector is that of charges at each node. Therefore, the matrix can be inverted and stored at the beginning of the calculation, which significantly reduces the computational time compared to 50 iterations used per time step for the finite difference method.

4. Results

Calculations have been made for air at a pressure of 760 Torr between parallel plane electrodes at different voltages, gap lengths and photoemission coefficients.

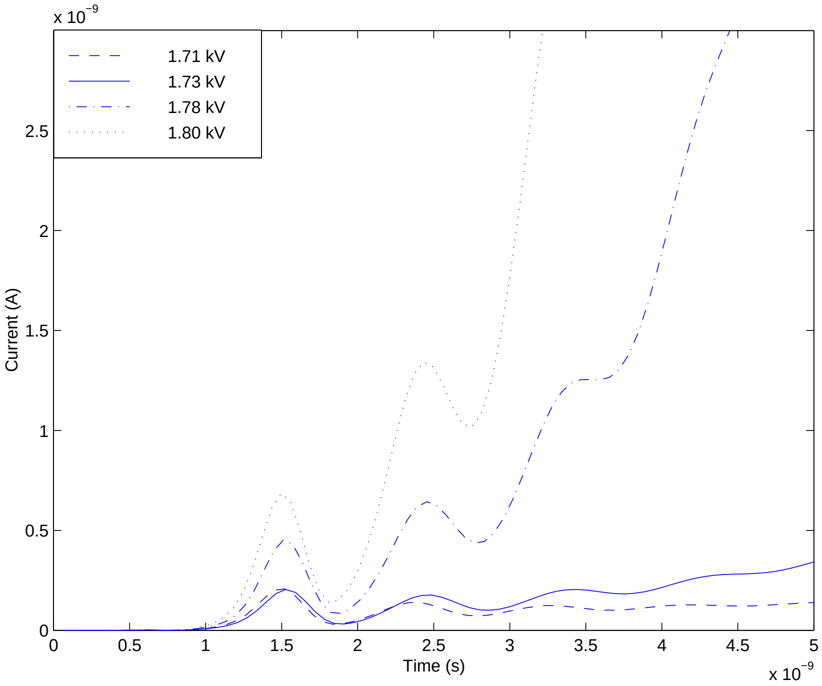

Figure 2. Current waveforms observed in a 0.025 cm parallel plane gap at voltages around the breakdown voltage.

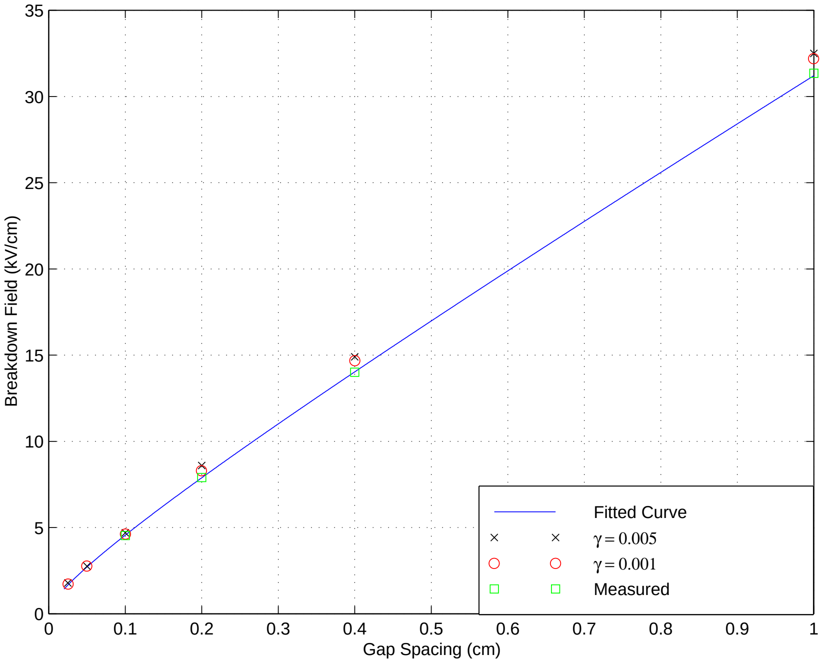

Figure 3. Breakdown voltage versus gap length, predicted by FE-FCT model and experiments.

The discharge is initiated by a pulse of ultraviolet light, which releases one electron from a small area of the cathode. If the electric field is high enough, this electron will collide with gas molecules knocking off new electrons, which in turn accelerate and create what is known as an electron avalanche. Up to this point the electric field in the gap is assumed to be unaffected by the accumulations of both electrons and ions, owing to the preceding generations of avalanches. When the electron avalanche becomes 'critical', in other words the space-charge accumulation is adequate to distort the field appreciably, the streamer begins to develop and the current rises abruptly. Figure 2 shows the current obtained in a 0.025 cm gap when different voltages in the region of the static breakdown voltage are applied. If the voltage is lower than the breakdown value (figure 2, 1.71 kV curve), successive avalanches are observed. But insufficient secondaries are produced to give a self-sustained discharge so the current steadily decreases. Above the threshold, the current rises slowly, due to space-charge avalanches until space-charge effects become apparent and

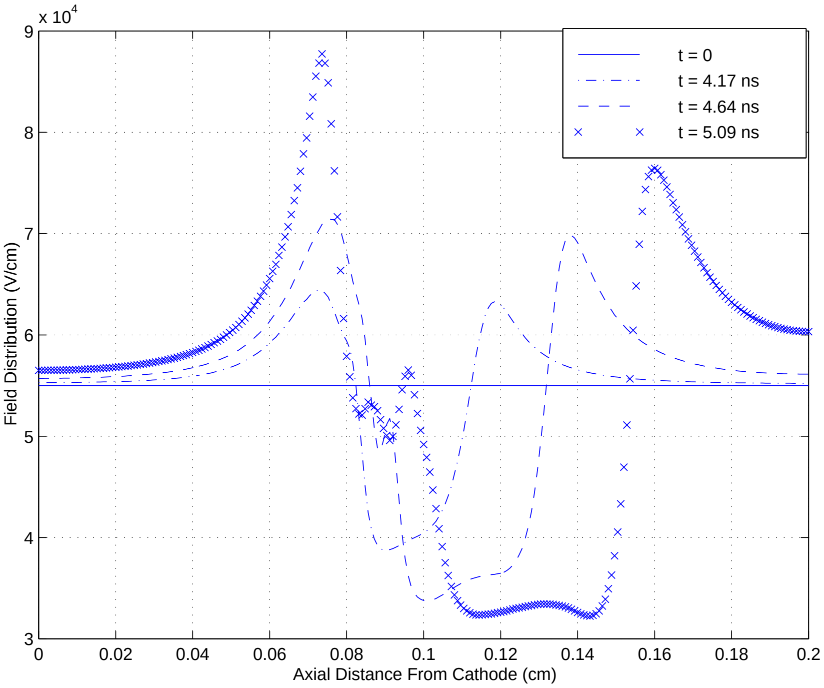

Figure 4. Field distribution along the axis of symmetry for a 0.2 cm gap at an applied voltage of 11 kV: first stage, critical avalanche.

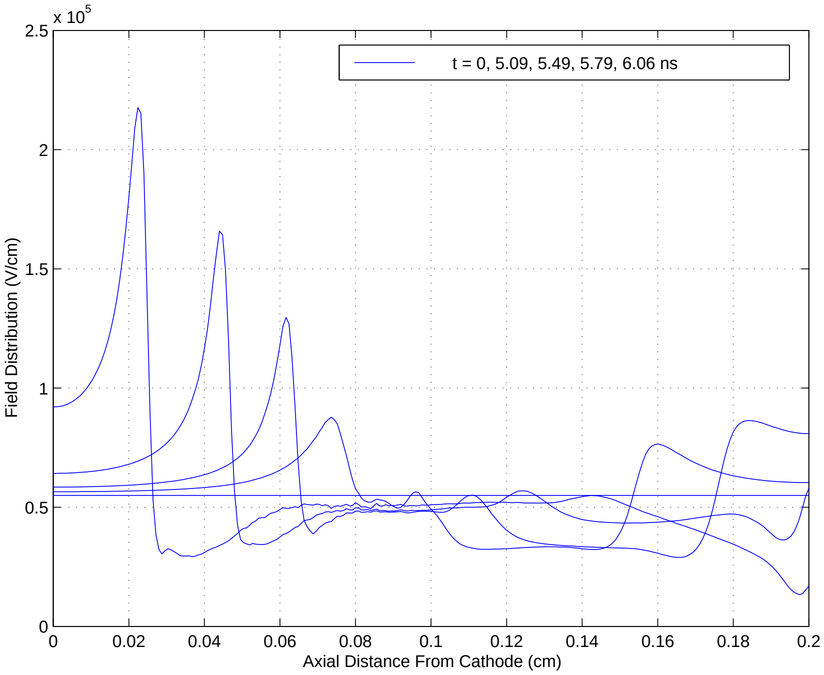

Figure 5. Field distribution along the axis of symmetry for a 0.2 cm gap at an applied voltage of 11 kV: streamer development stage.

the avalanche develops to a cathode and anode directed streamers with a very fast rise of current (figure 2, curve 1.78 kV, where the streamer forms after three successive avalanches). If the voltage is increased even further the space-charge accumulates even faster and the streamer occurs at earlier times (figure 2, curve 1.8 kV, which shows the streamer forming after two avalanches). Each peak in the current is caused by a group of electron avalanches reaching their maximum size just prior to absorption into the anode. The maximum current additionally implies that a maximum number of secondary electrons will be released at the cathode via photoemission, leading to another current maximum. The time between two successive avalanches obtained, agrees with the electron transit time (≈1 ns). The same current waveforms were observed experimentally by Raether [9].

The breakdown voltage is then predicted for different gap lengths. The voltage for a given gap is varied until breakdown occurs. Simulations were run for gaps in the range

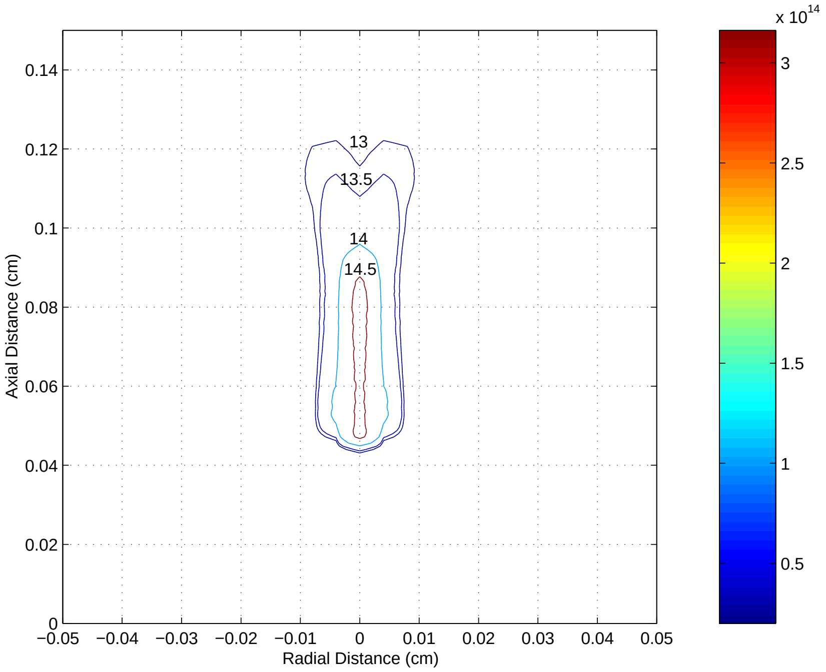

Figure 6. Contours of electron charge density during streamer propagation for a 0.2 cm gap at an applied voltage of 11 kV at time $t = 5.79$ ns. Contours are at densities: 10^{13}, $10^{13.5}$, 10^{14} and $10^{14.5}$ cm^{-3}.

of 0.025–1 cm with two different photoemission coefficients, γ, listed extensively in the literature: 5×10^{-3} [11] and 1×10^{-3} [5]. The voltage was varied in 0.01 kV increments in gaps of length up to 0.05 cm and in increments of 0.1 kV in gaps up to 1 cm. The predicted breakdown voltage is compared with experimental observations and the analytical expressions fitted to the experimental results given by Meek and Craggs [7]:

$$V_b = 25.5d + 6.6\sqrt{d} \qquad (6)$$

where V_b is the breakdown voltage in kV and d the gap length in cm.

The results, illustrated in figure 3, show very good agreement between computations and experiments with both photoemission coefficients with maximum error of 4% at a gap of 1 cm, demonstrating the accuracy and validity of the model. The errors become larger for longer gaps, due to the fact that photoionization is a necessary inclusion for longer gaps, in order to provide the seed electrons for the streamer propagation.

Figures 4 and 5 illustrate the field distribution along the axis of symmetry in a 0.2 cm gap when a voltage of 11 kV, 40% overvoltage is applied, at the avalanche-to-streamer transition and during cathode and anode directed streamer propagations, respectively. The development and propagation of anode directed and cathode directed streamers, as it is observed experimentally, is apparent. The critical distance for the development of a streamer is given by $\alpha/20$, where α is the ionization coefficient and is found to be 0.1 cm from the cathode, which agrees well with the results obtained (figure 4). The anode streamer propagates without any delay or any means of producing secondary electrons in front of it, as the electron drift produces the necessary electrons. However, the cathode directed streamer experiences a delay to its propagation, due to the time required for an adequate number of electrons to travel to the critical avalanche region, in order to feed the discharge with sufficient electrons for the cathode streamer to propagate. In sufficiently long gaps, the delay may cause very strong gradients, which can lead to instabilities and an electron shock wave [1]. In these instances, photoionization and thermal diffusion need to be included. Finally, electron charge density contours during the propagation of the cathode streamer are shown in figure 6 and the filamentary nature of the streamer is observed.

5. Conclusions

The new FE-FCT method is applied in its full two-dimensional form to the development of streamers and streamer propagation in atmospheric pressure air between parallel plane electrodes.

The method was verified by predicting very similar breakdown values to experimental results for different gaps and for the first time, the successive avalanche-to-streamer transition current waveforms, observed experimentally by Raether, have been produced numerically. Furthermore, the method correctly predicted the development and propagation of anode and cathode directed streamers in small parallel plane gaps when an overvoltage is applied.

The significant reduction in the number of unknowns, together with the ability to model complex geometries, make this algorithm a very good candidate for complex gas discharge applications.

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