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Measurement of the time-evolution of averaged impedances in a small atmospheric pressure spark-gap

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Abstract. The results of measurements of the time-evolution of averaged values of the resistance and inductance of a small, self-breaking atmospheric pressure spark-gap are presented. The gap lengths were varied in the range 0.4–4 mm, and the driving circuit used for the measurements was based on a 37 nF capacitor. Both the resistance and the inductance were found to diminish with time, in qualitative agreement with a radially expanding model for the spark channel.

1. Introduction

Spark-gaps are widely employed as switches in many fast electrical discharge circuits, such as plasma-producing devices, pulsed lasers and Blumlein lines. For a proper understanding and design of these circuits, a good knowledge of the time behaviour of the spark-gap impedance (that is, its resistance R_s and inductance L_s) is sometimes required, as pointed out in a previous work [1].

Time-averaged measurements of the values of R_s and L_s of a small, self-breaking spark-gap (gap lengths between 0.34 and 2 mm) were presented by Gratton *et al* [2], using a fast ringing circuit formed by a 2.12 nF capacitor charged at voltages ranging approximately from 2 to 11 kV. The inductances of the circuit were carefully minimized, so that the typical duration of the discharges were of the order of 50 ns, and the obtained R_s values were in the range 0.3–1 Ω.

In order to investigate the influence of the stored energy on the spark-gap impedance, we measured R_s and L_s using a circuit with a larger capacitor and a larger inductance so that the peak current values were approximately the same as those used by Gratton *et al.* Naturally the durations of our discharges were correspondingly larger, and in what follows we will describe our results and discuss some of their implications.

2. Experimental set-up

The experimental set-up used for our measurements is a very simple one, and it is sketched in figure 1. A plane parallel plate capacitor (about 1.6 m × 1 m aluminium plates separated by four 0.1 mm thick polythene foils), with a capacity $C = (37.0 \pm 0.5)$ nF, is charged through a 15 MΩ resistor up to the breakdown voltage V_0 of a spark-gap connected at one end of the capacitor, which has an adjustable separation d between its electrodes. The cathode of the spark-gap is constructed in brass and has a hemispherical shape with a 4.0 mm radius, while the anode is planar and made of aluminium. The electrodes system is completely open to the atmosphere. The total stray inductance and resistance of the circuit (excluding the gap) were measured with an auxiliary pulse generator and resulted in $L_0 = (8.4 \pm 0.3)$ nH and $R_0 = (27 \pm 4)$ mΩ, respectively. The system can operate with repetition rates between one and ten discharges per minute.

The gap lengths were chosen in the range 0.4 to 4.0 mm, and were measured using calibrated spacers with an uncertainty of ±0.1 mm. The corresponding V_0 values, measured with a standard resistive voltage divider, ranged between (2.7 ± 0.1) kV and (13.3 ± 0.1) kV (see table 1). During the discharges, the time derivative of the current, dI/dt, was measured by means of a

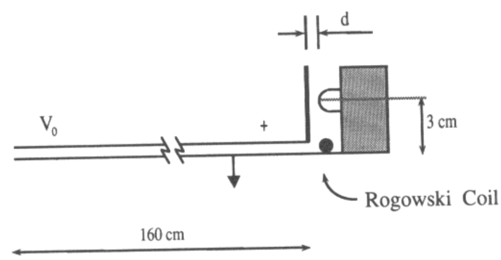

Figure 1. Schematic diagram of the experimental device used for the measurements.

Table 1. Measured values of the self-breakdown voltages V_0, gap lengths d and dI/dt rise time τ.

V_0 (kV)	d (mm)	τ (ns)
2.7 ± 0.1	0.4 ± 0.1	8.5 ± 1.0
3.9 ± 0.1	0.8 ± 0.1	9.5 ± 1.0
5.5 ± 0.1	1.2 ± 0.1	9.5 ± 1.0
7.1 ± 0.1	1.6 ± 0.1	11.0 ± 1.0
7.9 ± 0.1	2.0 ± 0.1	11.0 ± 1.0
9.0 ± 0.2	2.4 ± 0.1	13.5 ± 1.5
10.6 ± 0.1	2.8 ± 0.1	19.5 ± 2.0
11.4 ± 0.1	3.2 ± 0.1	20.5 ± 2.0
13.3 ± 0.1	4.0 ± 0.1	22.0 ± 2.5

Rogowski loop, and photographically registered on an oscilloscope, both at 20 ns per division and at 50 ns per division.

3. Results

In figures 2(a) and (b) we show, as example, typical 50 ns per division oscillographs obtained at $d = 0.4$ mm, $V_0 = 2.7$ kV and $d = 4.0$ mm, $V_0 = 13.3$ kV, respectively. We want to stress the fact that, in all cases, photographs were obtained by superimposing on each picture several consecutive discharges (from 6 to 15) performed under the same conditions. All of the registered pictures present only one clear trace, which indicates that the phenomenon is very reproducible.

Inspection of the waveforms shows that the first maximum of the signals departs from the ideal damped sinusoid one would expect to find. On the one hand, the profiles are flatter than those of sinusoid; on the other, their amplitudes are smaller (note that, in the oscillograph of figure 2(a), the amplitude of its first maximum is even smaller than that of the following minimum). This type of behaviour is frequently observed in fast discharge circuits, and is due to the finite closure-time of the 'switch', as has been discussed theoretically by Bruzzone *et al* [3], using a very simple model to describe the switching action. During this closure-time, R_s diminishes from a very large value down to a low one, and the circuit can perform as a damped oscillator only after the condition $R_s < 2(L/C)^{1/2}$ is satisfied. The distortion observed in this work essentially means that this condition is still not satisfied for a few tens of nanoseconds after dI/dt has reached its first maximum.

A significant parameter of this initial stage is the rise-time τ of the signals (the time interval required for dI/dt to reach its first maximum), which was found to depend on the gap length d. In table 1, the values of V_0, d and τ obtained from our experiment are listed. As a consequence of the initial behaviour of the spark-gap, the amplitudes of its first maximum and (to a lesser extent) the periods of oscillation measured from its time position are not reliable. Therefore, in what follows, these first maxima will be excluded from our analysis.

After this initial stage, the signals behave approximately as damped sinusoids, but with slight decreases in the values of the periods. This indicates that time-varying impedances are present in the circuit, their variations being, however, sufficiently gentle that we can safely take them as constants during each period. Under this assumption, we have determined from the registered curves mean values (on each period) of the total circuit inductance, $\langle L \rangle$ and resistance, $\langle R \rangle$, for consecutive periods displaced one from another by half a period. From them, we have evaluated the mean values of the spark inductance, $\langle L_s \rangle = \langle L \rangle - L_0$ and resistance, $\langle R_s \rangle = \langle R \rangle - R_0$, for all of our data.

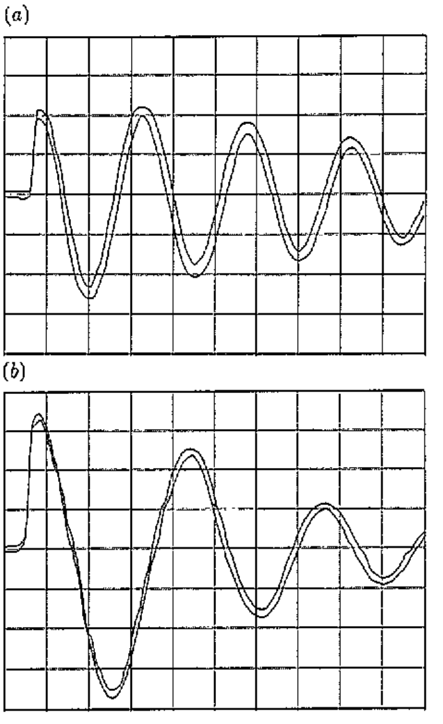

Figure 2. Oscillograms of dI/dt (50 ns per division) for (a) gap length 0.4 mm and $V_0 = 2.7$ kV and (b) gap length 4.0 mm and $V_0 = 13.3$ kV. The two curves shown in each figure are intended to represent the trace width.

In figures 3(a) and (b), we show the temporal evolution of $\langle L_s \rangle$ and $\langle R_s \rangle$ for each of the employed d values, excepting $d = 0.4$ mm because the uncertainties are in this case too large. As expected, at comparable instants of time, the values of both $\langle L_s \rangle$ and $\langle R_s \rangle$ are larger for larger values of d. It can also be seen that, for all the d values, both magnitudes monotonically decrease with the development of the discharge. Some caution should be exercised, however, with the observed diminution of $\langle R_s \rangle$. In fact, the presence of a diminishing inductance in the circuit, no matter how gentle is this variation, appears to the whole circuit as an added negative resistive term, d$\langle L_s \rangle$/dt. Order of magnitude estimations of this effect in our measurements yielded in some cases values of d$\langle L_s \rangle$/dt comparable to the observed $\langle R_s \rangle$ diminutions, so that the $\langle R_s \rangle(t)$ curves should be taken only as indicative of the resistance behaviour.

As $\langle L_s \rangle$ is expected to depend linearly on d, we have also evaluated $\langle L_s \rangle/d$, and the results are plotted in

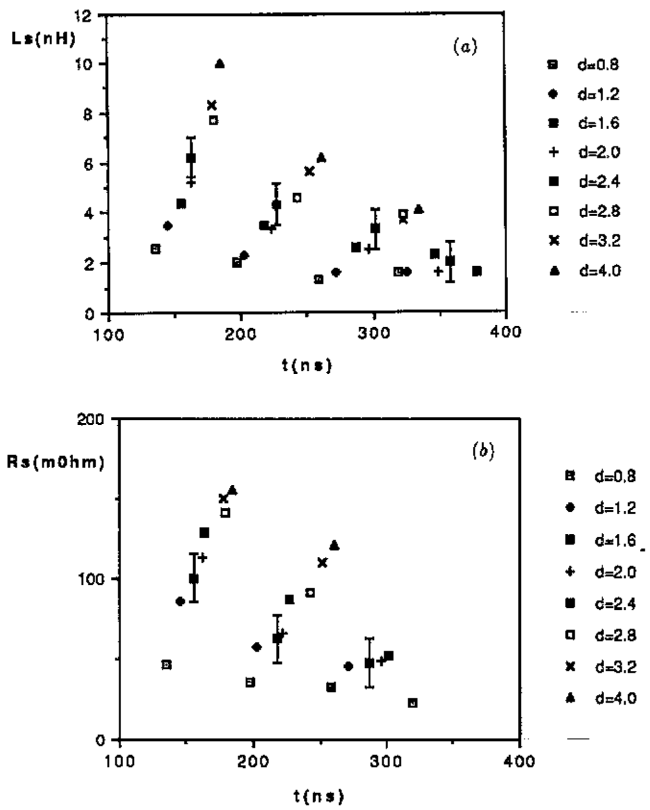

Figure 3. Temporal evolution of the spark inductance (*a*) and resistance (*b*), for several gap lengths.

figure 4 as a function of time (the case $d = 0.8$ mm was not included because of its large uncertainty). It is apparent that this magnitude also decreases monotonically with the discharge evolution, and is independent of the gap length. The coalescence of curves with different d values into a single one is rather surprising, and will be addressed later on. A plot of $\langle R_s \rangle / d$ versus t yields a similar behaviour, but we have chosen not to present it in view of the uncertainties associated with the $\langle R_s \rangle$ values discussed above.

4. Analysis of the results

The initial phase of the discharge corresponds to the breakdown of the gas in the gap (that is, the phase during which R_s swings from a very high value to a low one, lesser than that of the characteristic impedance in the circuit). Its duration has been studied, among others, by Martin [4], who called it the 'resistive time', τ_r. In his work, the following empirical relationship for the dependence of τ_r on the relevant parameters of the phenomenon is given:

$$\tau_r(\text{ns}) \approx 88(\rho/\rho_0)^{1/2} Z^{-1/3} E^{-4/3}$$

where ρ is the density of the gas used, ρ_0 the density of air at NTP, Z the impedance of the driving circuit in ohms and E the applied electric field in units of kV mm^{-1}. Using for the impedance of our circuit $Z = (L_0/C)^{1/2} \approx 480$ mΩ, and $E = V_0/d$, we have evaluated the values of τ_r corresponding to our data, and for comparison purposes, the measured values of τ and their corresponding τ_r are shown in table 2. It can be seen that the agreement of the two sets of times is reasonably good. In consequence, during the first peak in the dI/dt signals, the rapidly changing values of R_s are still of the order of the critical damping resistance of the circuit. Hence, this part of the signal should be distorted with respect to an ideal damped sinusoidal curve.

The reported behaviour of $\langle R_s \rangle$ and $\langle L_s \rangle$ during the subsequent stages of the discharge is compatible with the spark model described by Braginskii [5], who assumed the formation of an electrical conducting submillimetric cylindrical channel of radius a, supersonically expanding against the surrounding gas. More recently, Herziger *et al* [6, 7] have studied the dynamics of sparks in air at atmospheric pressure, and have shown experimentally that the initial value of the spark radius is $a = 50$ μm, when the applied electric field equals the breakdown field in the gap (which is the case in our measurements). It is clear that a channel whose radius increases during discharge evolution should have a time-decreasing inductance, and, if its electrical conductivity does not decrease with time, should also have a time-decreasing resistance. Theoretical predictions of the evolution of these magnitudes are possible, but require development of very complex codes, which are outside the scope of this work. We will limit the discussion to giving some simple estimations in order to show that, with the parameters employed in this work, description of the spark as an expanding channel is consistent.

To this end, an estimation of the ratio between

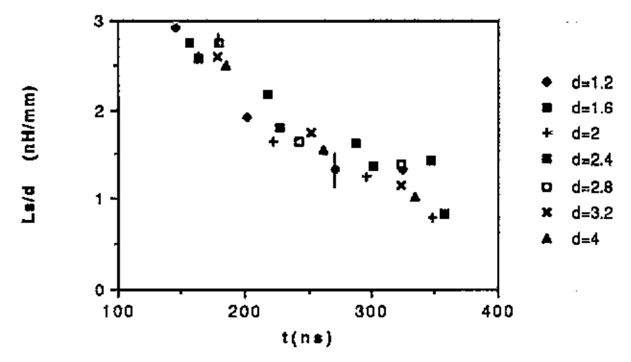

Figure 4. Mean values of the spark inductance per unit length, as functions of time, for several gap lengths.

Table 2. A comparison between the measured rise times and Martin's resistive time τ_r.

d (mm)	τ (ns)	τ_r (ns)
0.8 ± 0.1	9.5 ± 1.0	12.3 ± 1.6
1.2 ± 0.1	9.5 ± 1.0	13.3 ± 2.0
1.6 ± 0.1	11.0 ± 1.0	13.9 ± 1.6
2.0 ± 0.1	11.0 ± 1.0	16.3 ± 1.6
2.4 ± 0.1	13.5 ± 1.5	17.4 ± 1.7
2.8 ± 0.1	19.5 ± 2.0	17.1 ± 1.3
3.2 ± 0.1	20.5 ± 2.0	18.8 ± 1.2
4.0 ± 0.1	22.0 ± 2.5	20.5 ± 1.2

thermal and magnetic pressures at the early stage of the discharge is required. From figure 2, one can see that dI/dt, after an initial rise (essentially linear) during a time τ, remains nearly constant for at least 10 ns, at a value $(dI/dt)_0$. Let us assume that, at time $t=\tau$, a cylindrical channel is formed, with a degree of ionization sufficient to ensure that its electrical conductivity, σ, corresponds to that of a fully ionized plasma (a few percent, at least). We can estimate the time $\tau+\Delta\tau$ needed to complete the ionization, under the assumption that it will be so small that the energy delivered to the channel will be consumed essentially in the ionizing process, disregarding radiation and thermal conduction losses and also movements of the plasma. This process will take place at constant electron temperature (T_e), and hence at constant σ. Disregarding the initial ionization degree, we can write

$$\int_{\tau}^{\tau+\Delta\tau} R_s I^2(t)\,dt = n_0 \varepsilon V \qquad (1)$$

where $R_s = d/\pi a^2\sigma$; $I(t>\tau)=(dI/dt)_0(t-\tau/2)$; ε is the mean 'energetic cost' of an ionization; n_0 is the atomic density of air at atmospheric pressure (about $5\times10^{25}\ m^{-3}$) and the spark volume $V=\pi a^2 d$.

From (1) we obtain

$$2\,\Delta\tau = \left(\frac{24\pi^2 a^4 n_0 \varepsilon\sigma}{(dI/dt)_0} + \tau^3\right)^{1/3} - \tau$$
$$= (\tau_i^3 + \tau^3)^{1/3} - \tau. \qquad (2)$$

The parameter τ_i is a characteristic time of the process. By replacing constants, and using, as practical units, nanoseconds for τ_i; $5\times10^{25}\ m^{-3}$ for n_0; kA ns^{-1} for $(dI/dt)_0$; tenths of a millimetre for a; and electron-volts for both ε and T_e, we find

$$\tau_i \approx 0.83\,\frac{n_0^{1/3}\varepsilon^{1/3} a^{4/3} T_e^{1/2}}{(dI/dt)_0^{2/3}}. \qquad (3)$$

Taking $\varepsilon\approx18.3$ eV (ionization plus one half of the N_2 dissociation energy); $T_e\approx2$ eV; $(dI/dt)_0\approx0.2$–0.6 kA ns^{-1} (from our data) and using $a\approx50\ \mu m$ one finds τ_i values in the range 2–4 ns, which yield $\Delta\tau$ values smaller than 1 ns, sufficiently short as to justify the initial assumptions.

Admittedly, the evaluation of $\Delta\tau$ is rather crude. However, it is satisfactory to find that the dependences of the time constant τ_i, whose value rules this process, are weak on the unknown value of T_e, and weaker still on the values of ε, so that even in cases in which energy losses (radiation, formation of excited species, and so on) would be important, they could be accounted for by enlarging the ε value, without significant changes in the value of τ_i.

Once the ionization process has been completed, the energy equipartition time between electrons and ions (N^+) is, in the present situation, much smaller than 1 ns, so that at a time $\tau+\Delta\tau$, the conducting channel should be formed by a fully thermalized plasma. The thermal pressure, $3n_0kT_e$, amounts to $5\times10^7\ N\ m^{-2}$ (using again $T_e=2$ eV), while the magnetic pressure, $\mu_0[I(\tau+\Delta\tau)]^2/(8\pi^2a^2)$, reaches at most a value of $5\times10^4\ N\ m^{-2}$, three orders of magnitude smaller than the thermal pressure. Therefore, an expansion of the channel should follow, producing a cylindrical shock wave in the surrounding gas.

5. Final remarks and conclusions

Using very simple measurements, we have shown that the inductance and resistance of a small atmospheric pressure spark-gap decrease with development of the discharge, consistently with the picture of an expanding plasma channel.

It is reasonable to assume for an expanding cylindrical channel that $\langle L_s\rangle/d$ is proportional to ln[constant/$a(t)$]. Hence, the behaviour of $\langle L_s\rangle/d$ as a function of time in a given discharge reflects the particular $a(t)$ behaviour in that discharge. The fact that all d curves coalesce into a single one implies that the kinematics of these expansions are roughly the same for all the sparks having different gap lengths (that is, different stored energies and time periods), which is certainly not granted *a priori*. We believe this to be a fortuitous result of our experiment, which could be explained as follows. The relevant magnitude determining the expansion kinematics, at fixed gas nature and pressure, is the power delivered per unit length to the spark. In our measurements, the total energy available per unit spark length $(CV_0^2/2d)$ increases from 9.5 J m^{-1} at 1.2 mm up to 22 J m^{-1} at 4 mm, while the corresponding mean discharge periods also increase from 120 ns up to 160 ns. Hence, the power per unit length does not change very much with d in our experiment, and a fairly similar $a(t)$ behaviour can be expected for all cases.

A comparison between the values of $\langle R_s\rangle$ found in [2] and those presented in this work (at equivalent gas pressure and gap lengths) show that our $\langle R_s\rangle$ are systematically smaller. In fact from figure 5 of Gratton's work we find $\langle R_s\rangle\approx600$ mΩ for $d=2$ mm, and $\langle R_s\rangle\approx200$ mΩ for $d=1$ mm. These values were obtained from damped sinusoids having time-periods of around 15 ns (according to their figure 4). In our measurements, $\langle R_s\rangle$ at $d=2$ mm is always less than 130 mΩ, and at $d=1.2$ mm it is smaller than 80 mΩ. The stored energy per unit length is in our experiment about 18 times larger than that used by Gratton, but our time periods are also larger, roughly by a factor of ten. Hence, the values of the power per unit length used in this work are typically 1.8 times larger than those used in [2], and consistently the values of $\langle R_s\rangle$ are smaller.

In conclusion, our results suggest that the power delivered per unit length to the spark is the relevant magnitude which rules its impedance evolution.

Acknowledgments

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