Monte Carlo Simulations of Relativistic Runaway Electrons and Terrestrial Gamma Ray Flashes (TGF)

Nikolai G. Lehtinen

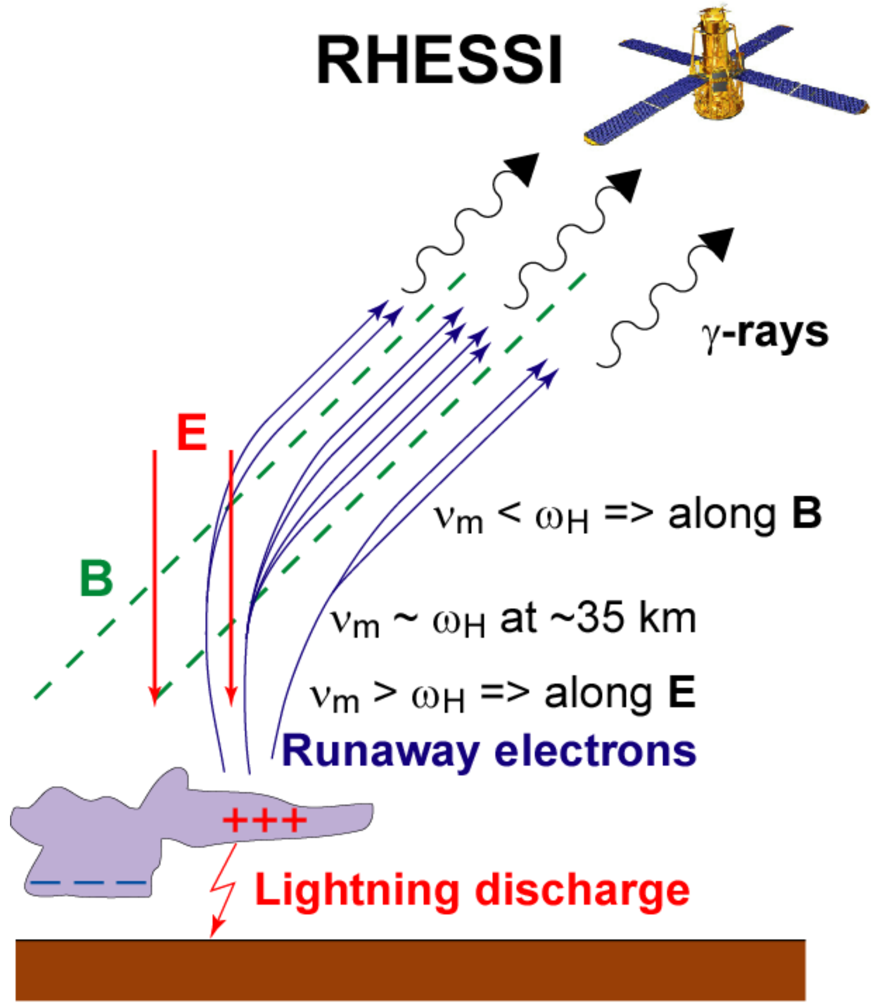
February 15, 2005

Terrestrial Gamma Flashes (TGF)

- ~1 ms duration
- Energies of up to 20 MeV
- Hard spectrum
- First observed by BATSE, ~1/month
- RHESSI detects ~10-20/month, => ~50/day globally

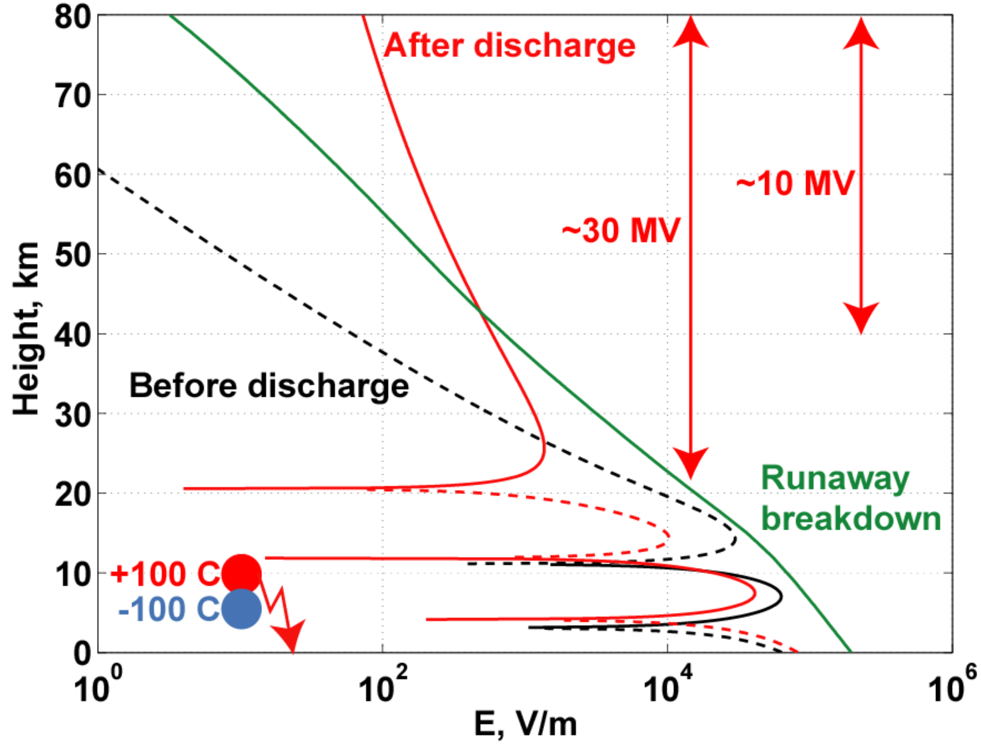
Bremsstrahlung mechanism

- Electrons are accelerated by E field following a +CG discharge
- Number of electrons is increased due to ionization
- Gamma rays are produces by bremsstrahlung from energetic electrons (~35 MeV energy)
- Emission is forward-directed
- Direction of electron motion is determined by elastic collision rate and B.



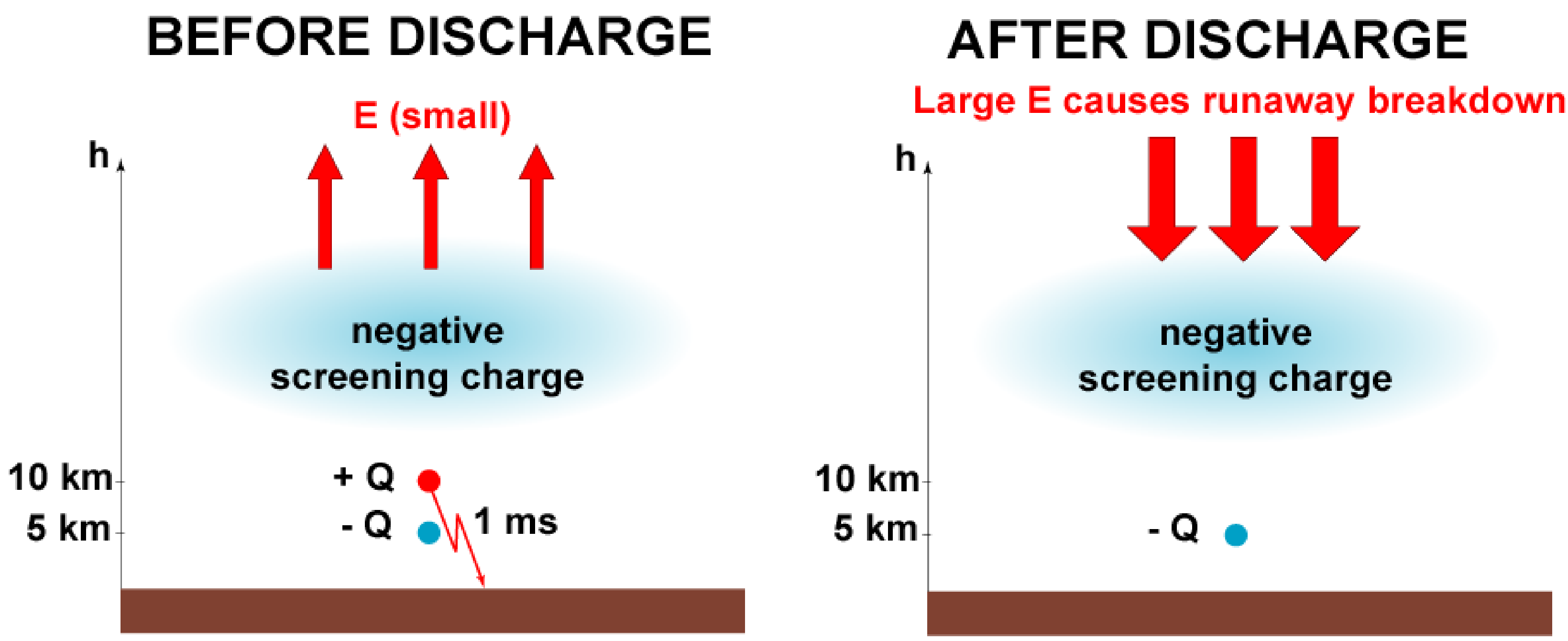
Post-discharge electric field

- Exceeds relativistic runaway threshold
- Accelerates electrons upward
- In atmosphere with exponential conductivity profile:

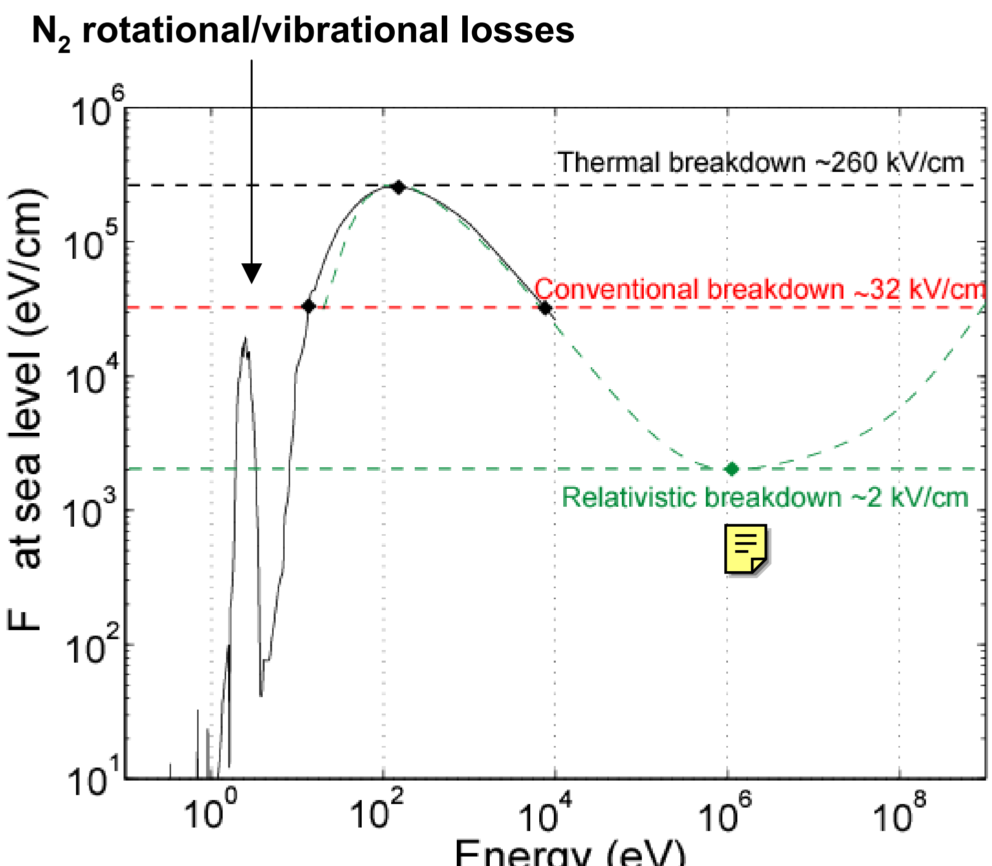


$$\Phi_{\mathrm{disch}} = \frac{Q}{4\pi\varepsilon_0} \frac{\exp[-(r+z)/2H]-1}{r}$$

Quasi-Electrostatic Field



Relativistic Runaway Mechanism and Dynamic Friction Force



$$F = \sum N\sigma_i(v)\Delta\varepsilon_i$$

43 inelastic processes:

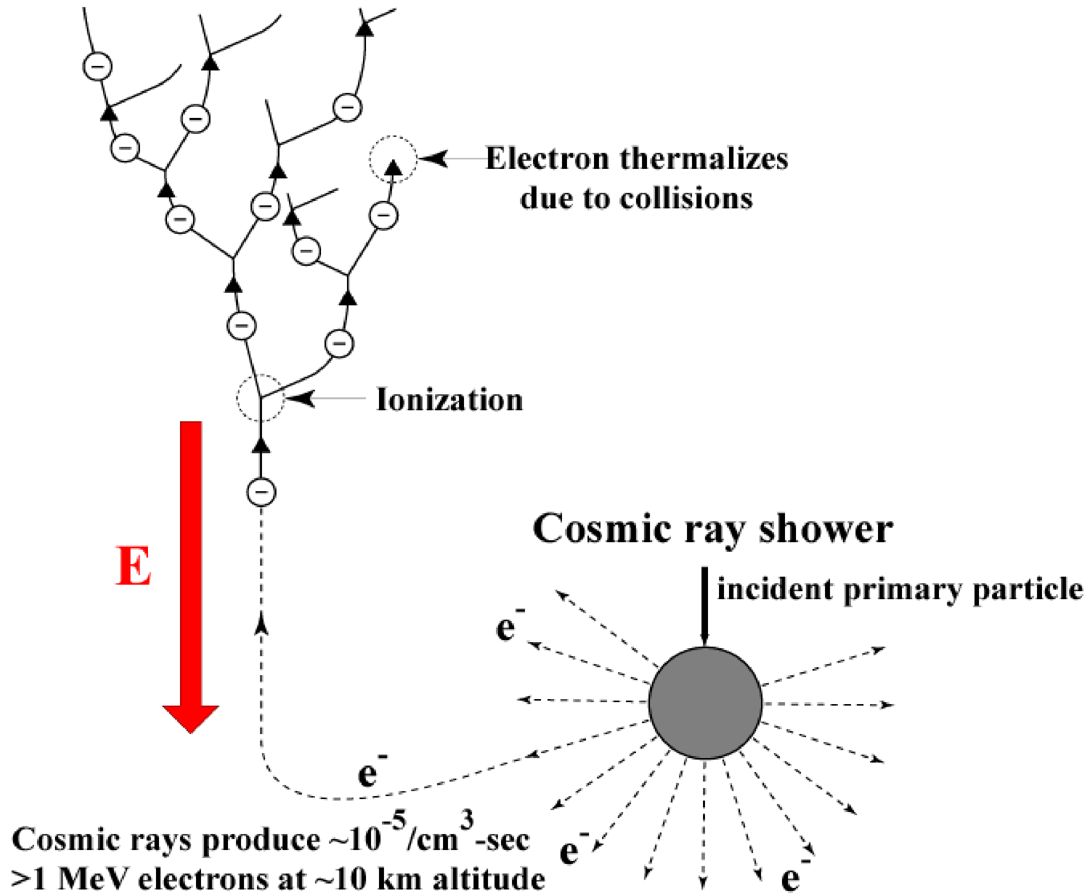
- **Rotational, vibrational, electronic level excitations**
- **Dissociative losses**
- **Ionization**

In townsends (1 Td = 10^{-21} V-m^2):

E/N = 8 Td, 130 Td, 1000 Td

Relativistic Runaway Electron Avalanche

- Cosmic ray primaries
- Ionization with production of relativistic electrons
- Acceleration



Monte Carlo simulations

Method:

- Relativistic motion

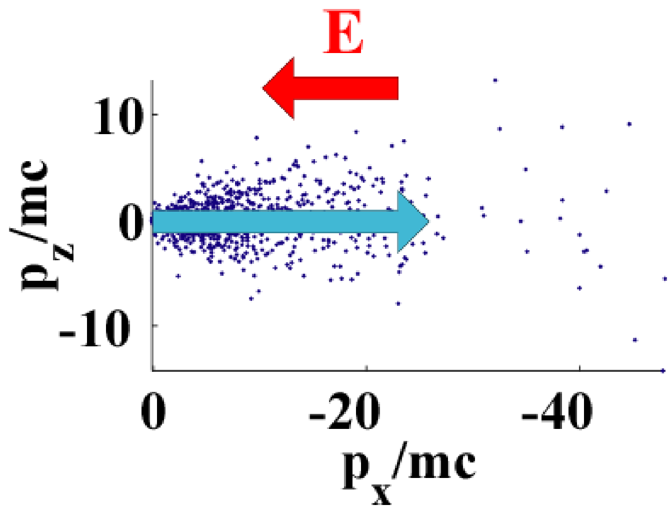
$$\frac{d\mathbf{p}}{dt} = -eE - \frac{e}{m\gamma}\mathbf{p} \times \mathbf{B} + \mathbf{G}(t)$$

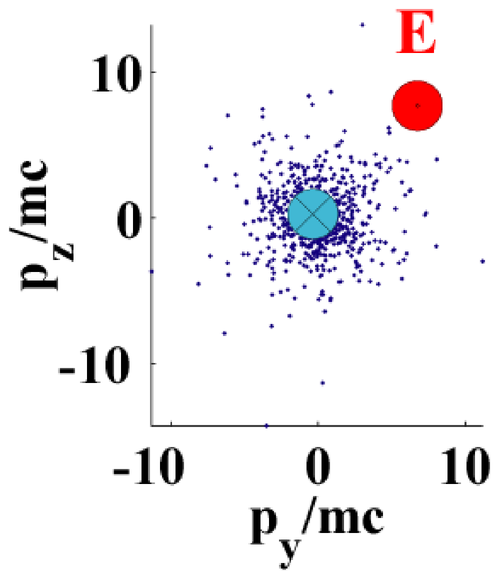
- G(t) includes inelastic energy losses and elastic scattering (but excludes energy losses from ionization)
- New electrons from ionization

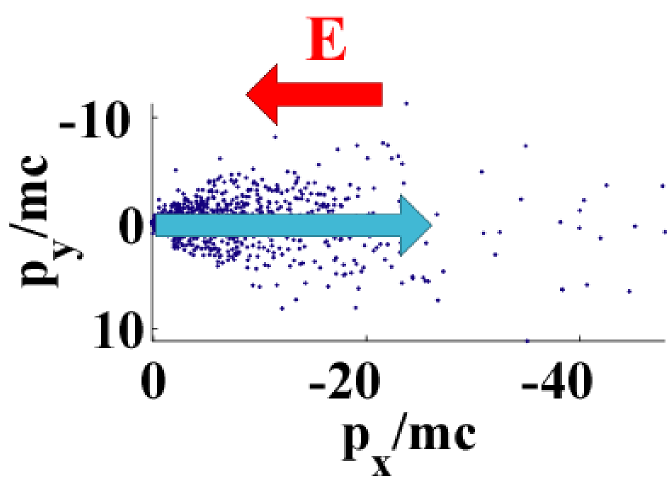
Results:

- Electron distribution
- Avalanche growth rate
- Drift velocity

Electrons in momentum space
$E = 5\ E_t,\ B = 0$







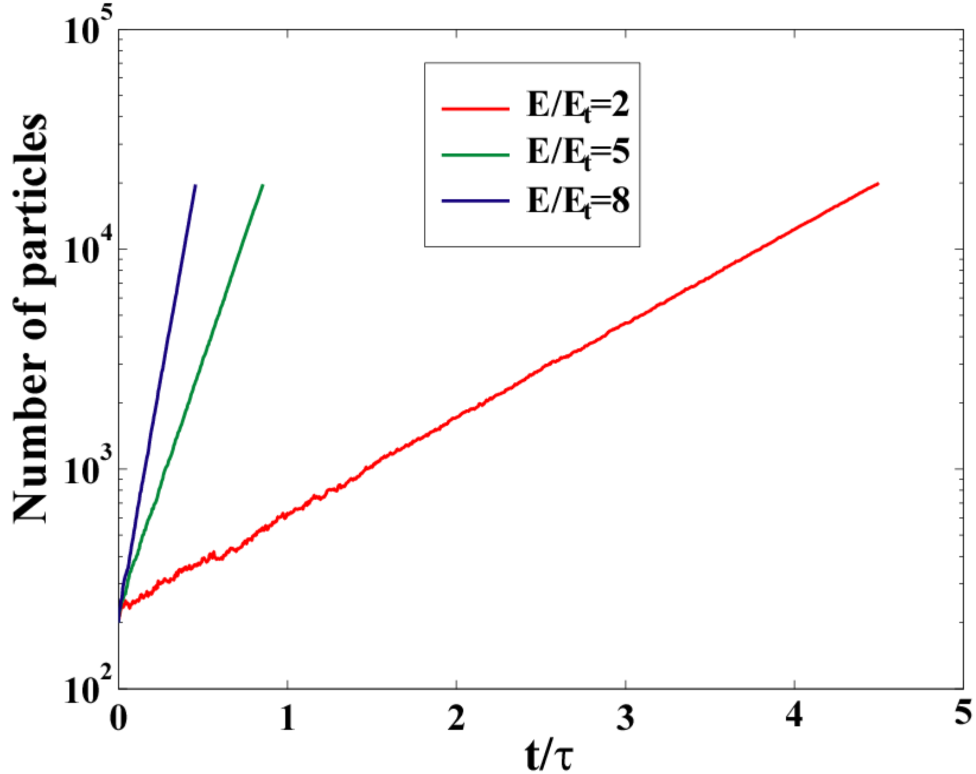
Growth rate

- $N = N_0 e^{Rt}$
- $R = R_0/\tau$
- Proportional to atmosphere density N_m

$$\tau = (2\pi N_m Z_m r_0^2 c)^{-1}$$

  - r_0 – classical electron radius
  - Z_m – molecule charge

- $R_0 \sim (\delta-1)+0.04(\delta-1)^2$, $\delta=E/E_t$



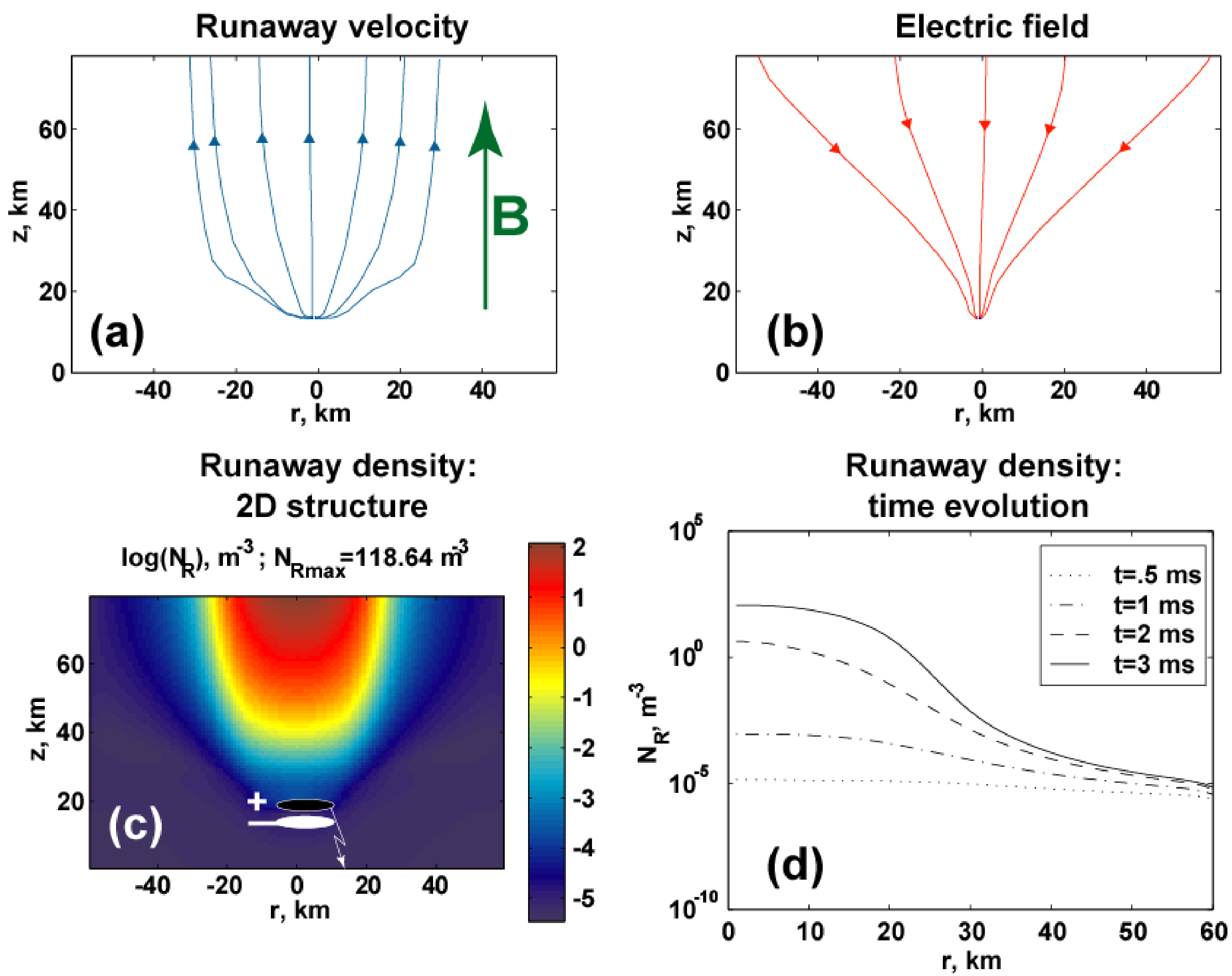
Fluid modelling of runaway avalanche above thundercloud

$$\frac{\partial N}{\partial t} + \nabla(vN) = RN + S$$

- **Notations:**
  - **v – drift velocity**
  - **$R(E/E_t)\sim N_m$ – avalanche rate**
  - **$S\sim N_m$ – source from cosmic rays, $=10\ m^{-3}\ s^{-1}$ at 10 km**
- **Cylindrical**
  - **Vertical magnetic field**
- **Cartesian**
  - **Arbitrary direction of magnetic field**
  - **Horizontally extended thundercloud**

Fluid modelling results: Cylindrical

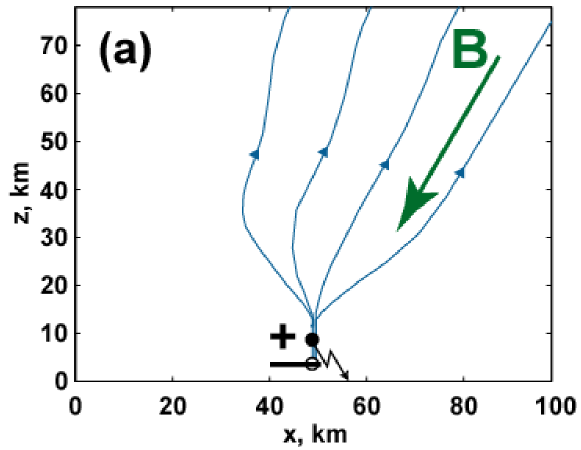
Q=420 C, h_+=20 km, t=3 ms



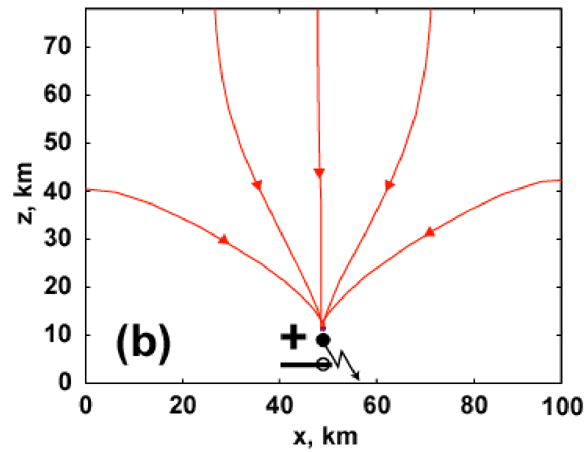
Fluid modelling results: Cartesian

Q=12 C/km, h_+=10 km, t=3 ms

Runaway velocity

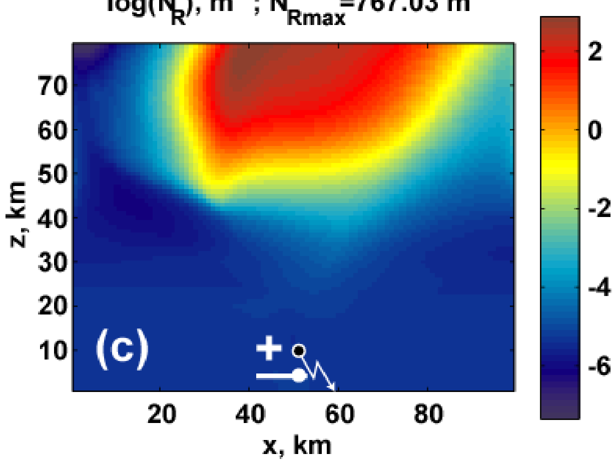


Electric field

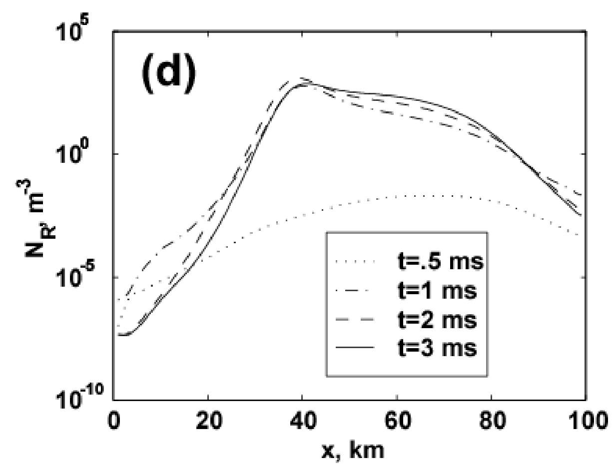


Runaway density: 2D structure

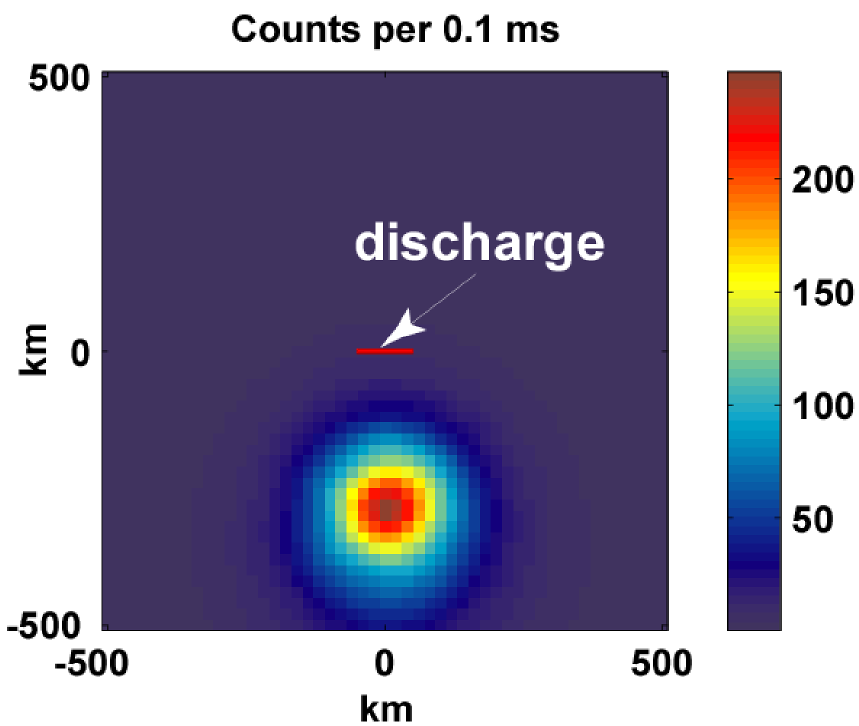
log(N_R), m^{-3}; N_{Rmax}=767.03 m^{-3}

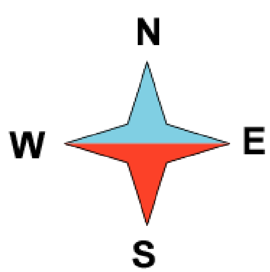


Runaway density: time evolution



Simulated BATSE photon counts



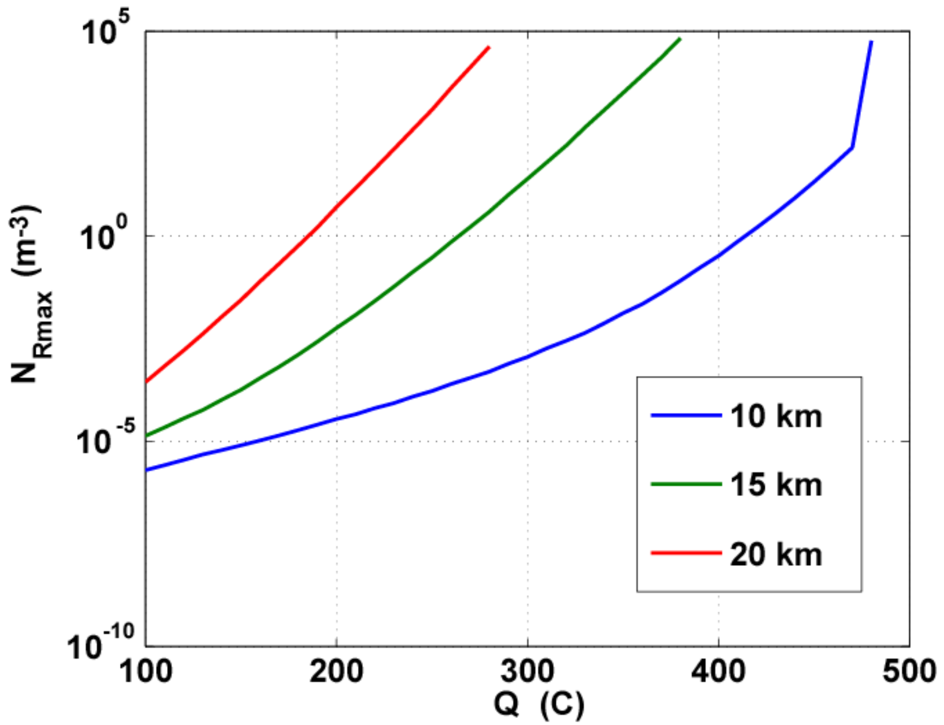


- Q=1200 C, 100 km long cloud (in EW direction)
- 45° N latitude
- 500 km altitude
- 100-300 keV
- Beam width ~ mc^2/E_{el}
- Spectrum is E_{ph}^{-1} on average, harder in the center, softer on the edges

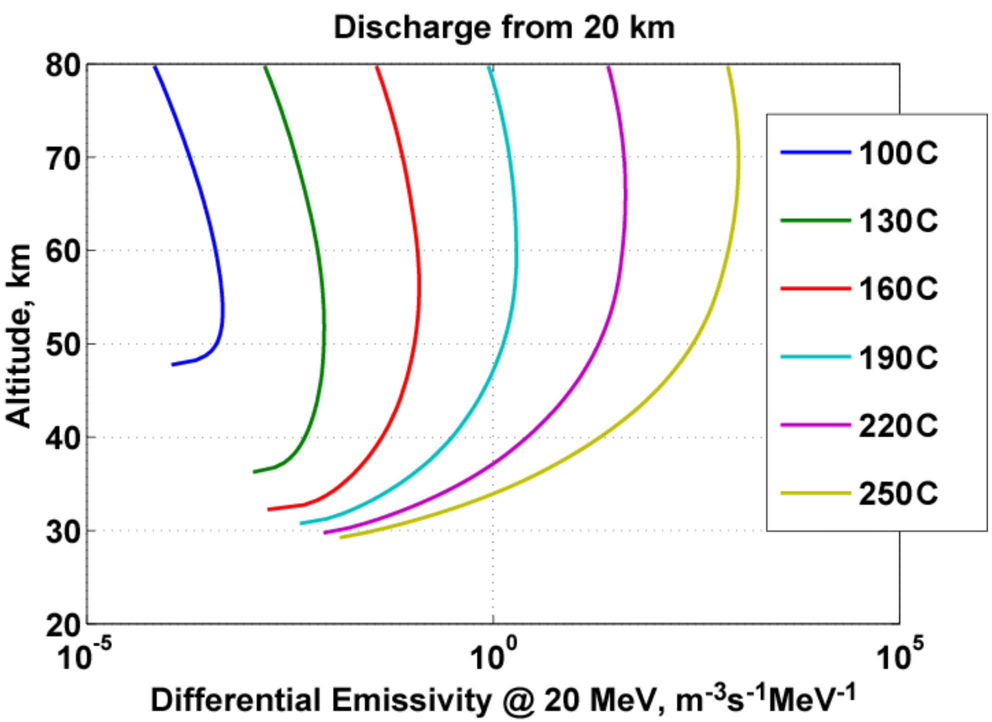
Highly nonlinear dependence on the charge removed and altitude

$$v\frac{dN}{dz} = RN + S$$

- v=0.9c – runaway drift velocity
- $R(E/E_t)$~N_m – avalanche rate
- S~N_m – source from cosmic rays, =10 m^{-3} s^{-1} at 10 km



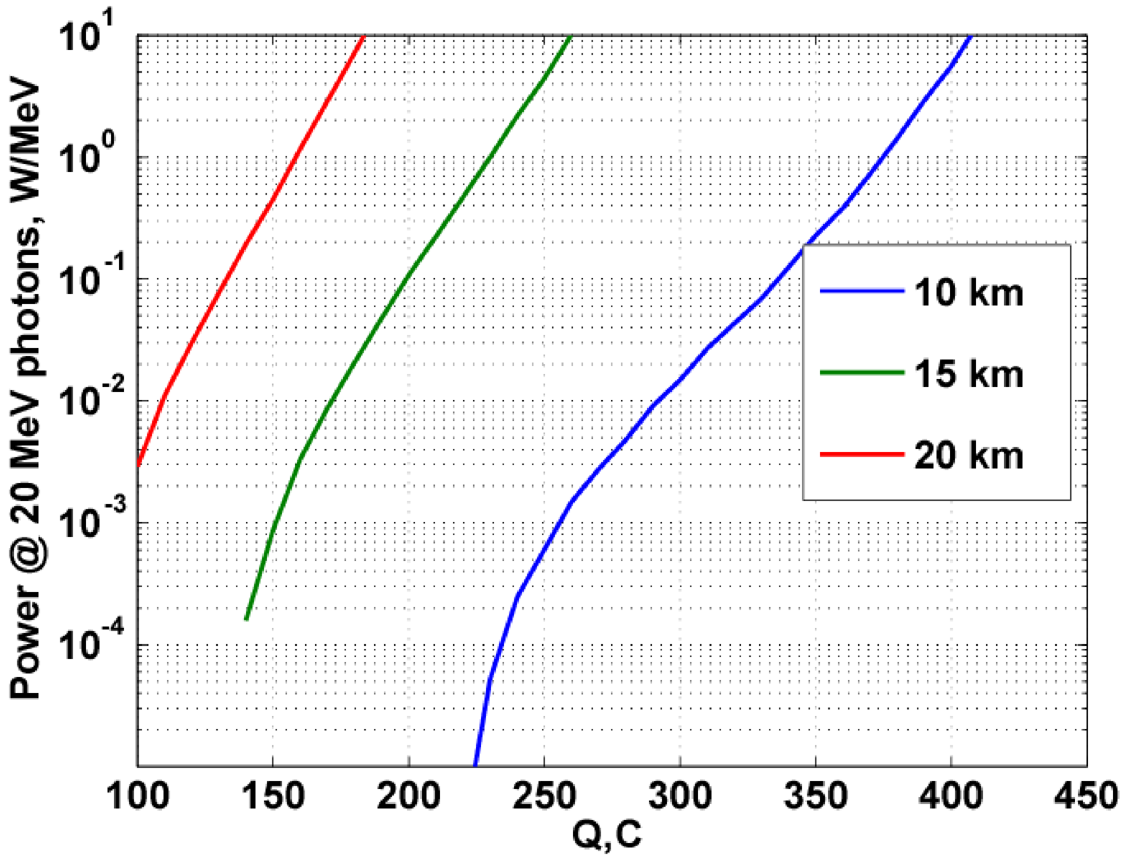
Gamma Ray Emissivity



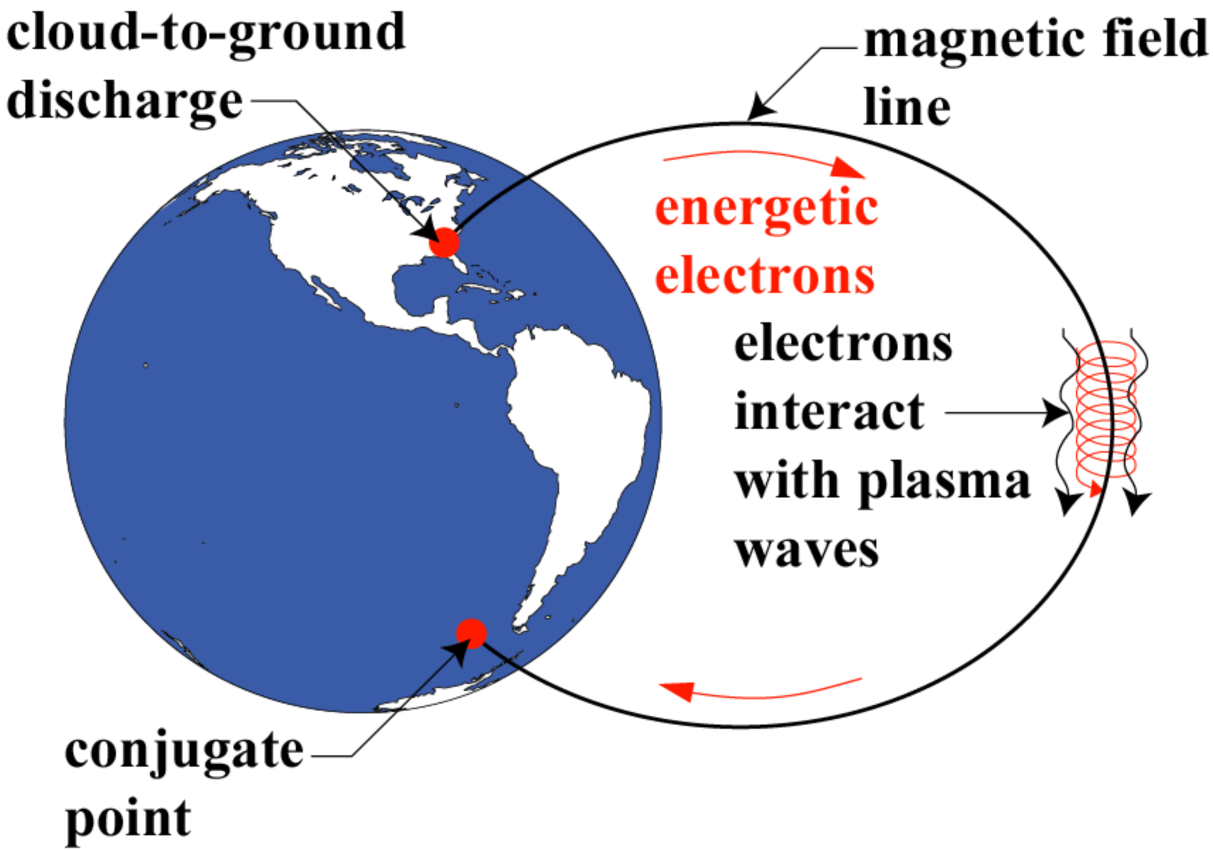
- $\varepsilon = N N_m v\, d\sigma/dE_{ph}$
- $d\sigma/dE_{ph}$ – Heitler's differential bremsstrahlung cross-section

The power produced in gamma rays

Integrate over volume, assume ~10km transverse size of the electron beam

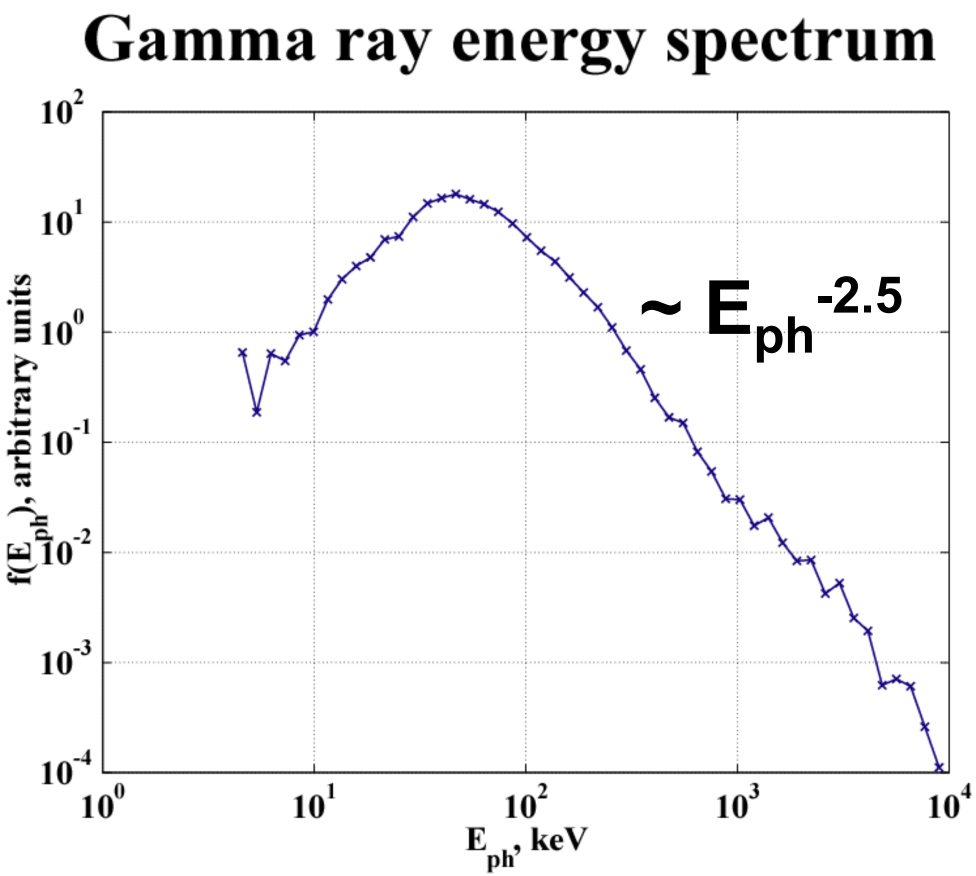


Escaping runaway electrons precipitate at the conjugate point



Gamma rays from precipitating electrons

- Initial gamma ray direction is downward
- They are backscattered
- Soft spectrum
- Observed spectrum is ~ E_{ph}^{-1}



Runaway Electron Avalanche and TGFs at the Magnetic Equator

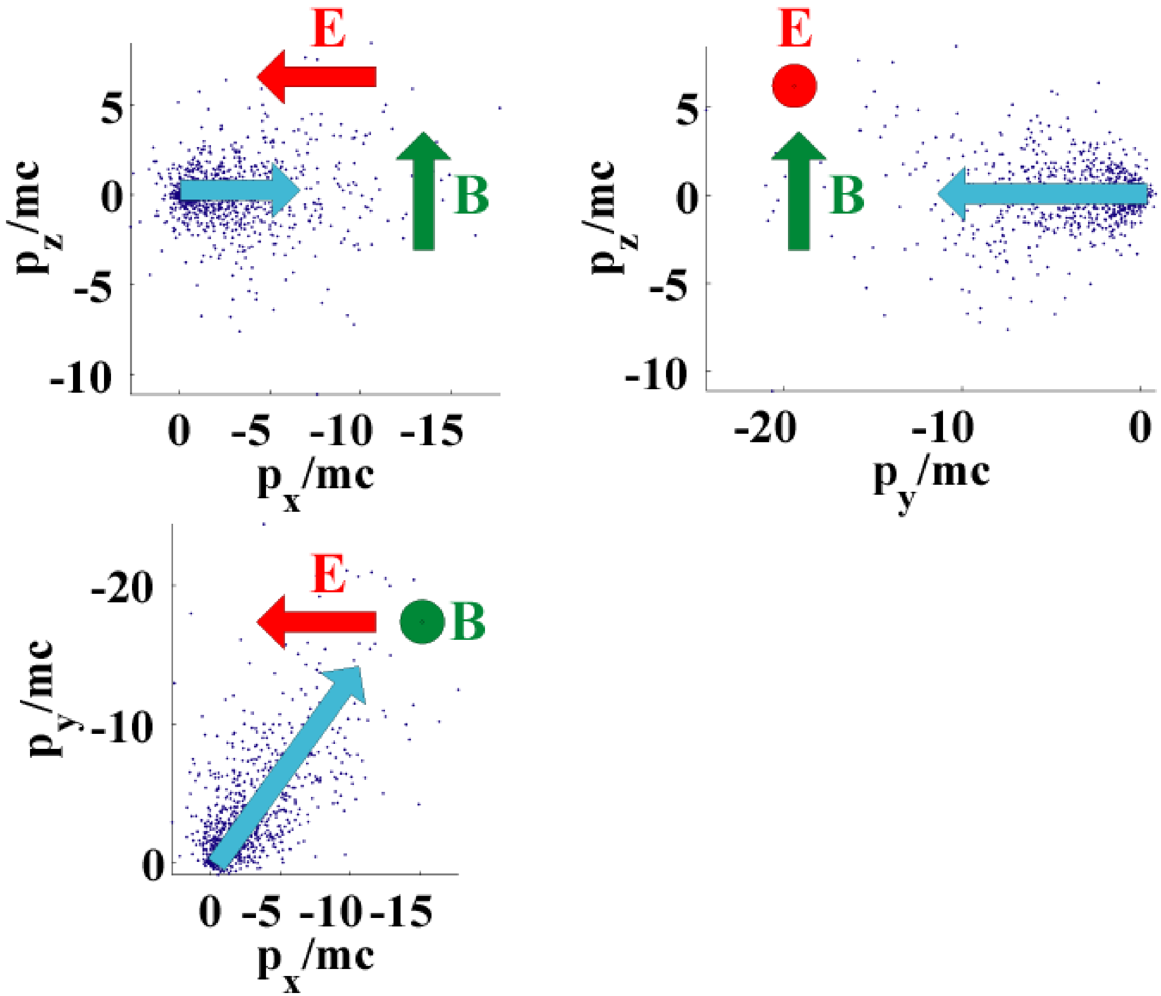
Relativistic Runaway Breakdown at the Equator

- Numerous RHESSI observations

Contradiction:

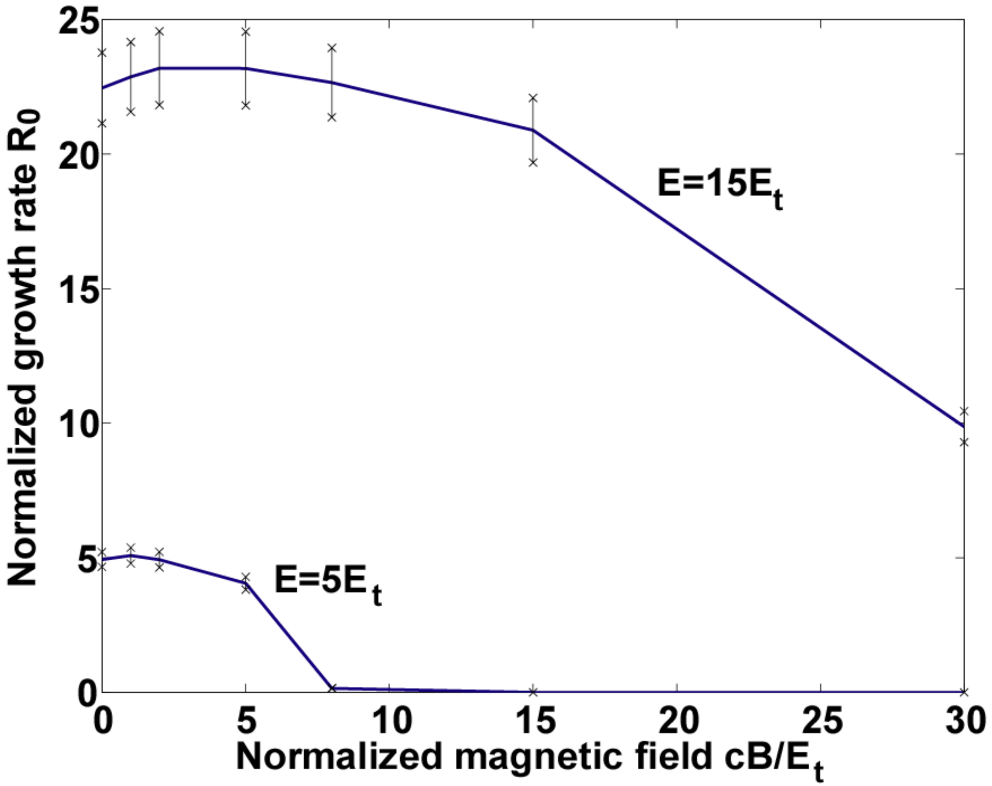
- Suppressed by geomagnetic field
- ExB drift in the horizontal direction => gamma rays are not emitted vertically?

Electrons in momentum space
$E = 5\,E_t$, $B = cE$, $B \perp E$



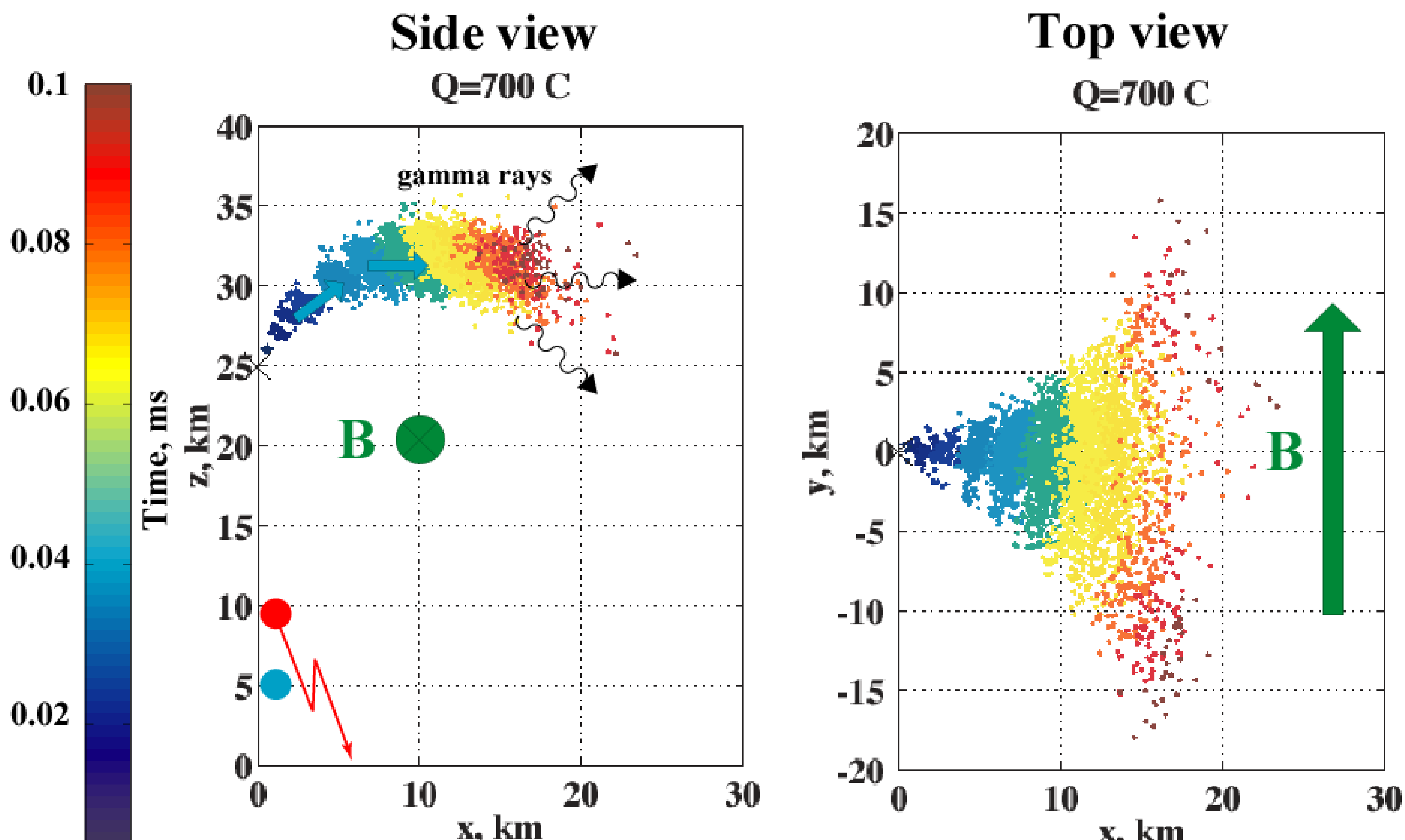
NOTE: There is no avalanche for $B \perp E$, $B=2cE$

The avalanche growth rates with perpendicular magnetic field



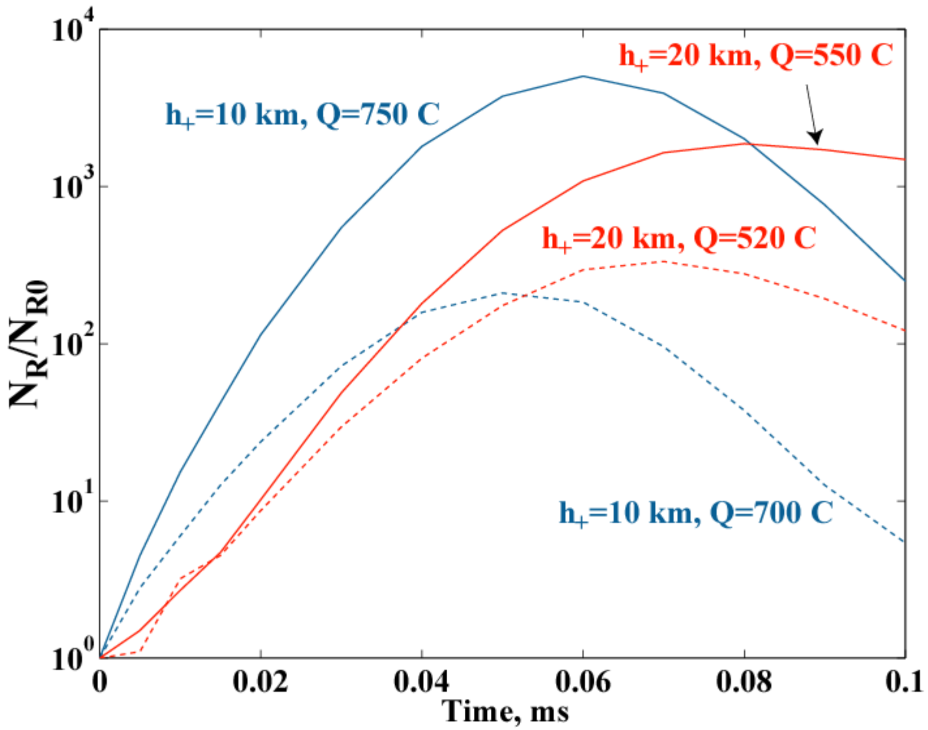
- The rate decreases slowly with increasing B at small B
- The avalanche is quenched approximately when B>2E/c

Nonuniform Monte Carlo results at the equator



Number of particles (MC simulation at the equator)

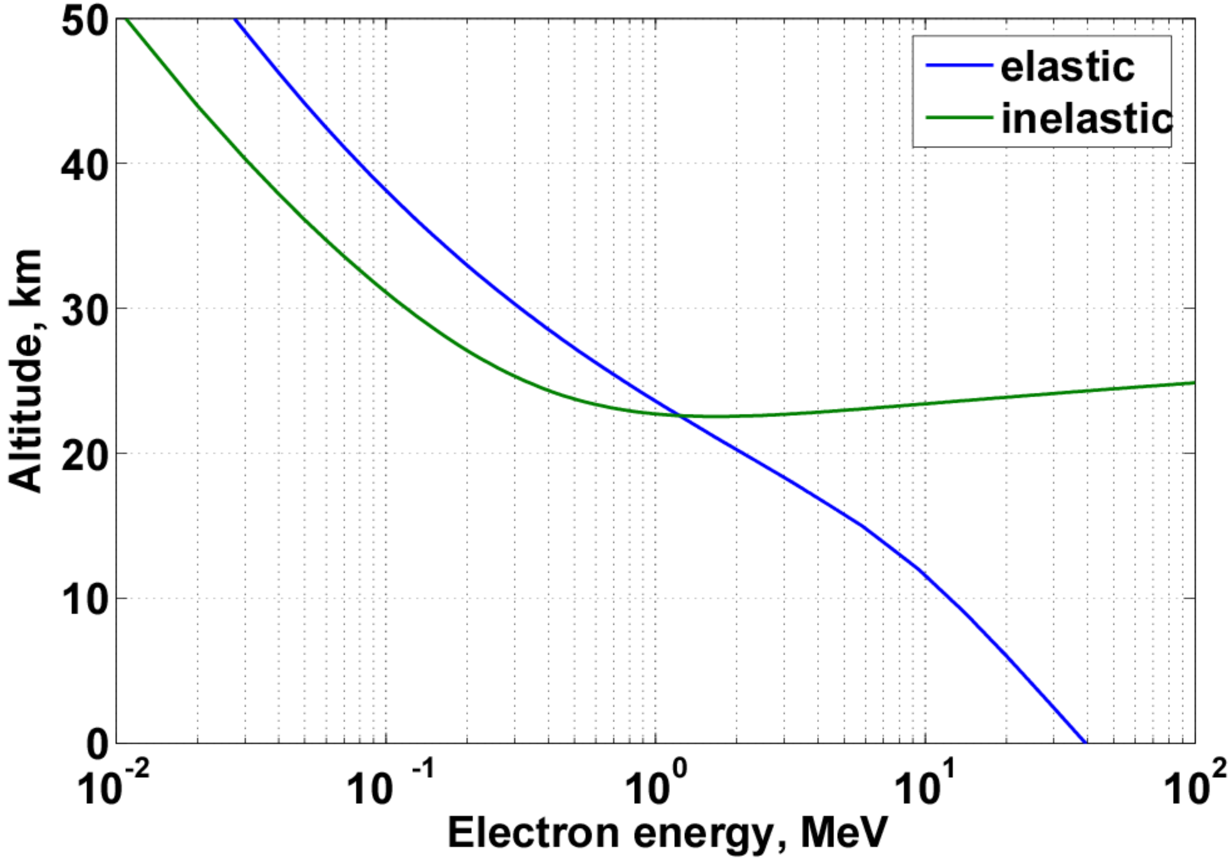
Number of particles with energy > 2 keV



- Higher charge removal needed
- Energetic electrons are lost quickly due to moving out of the high E field region

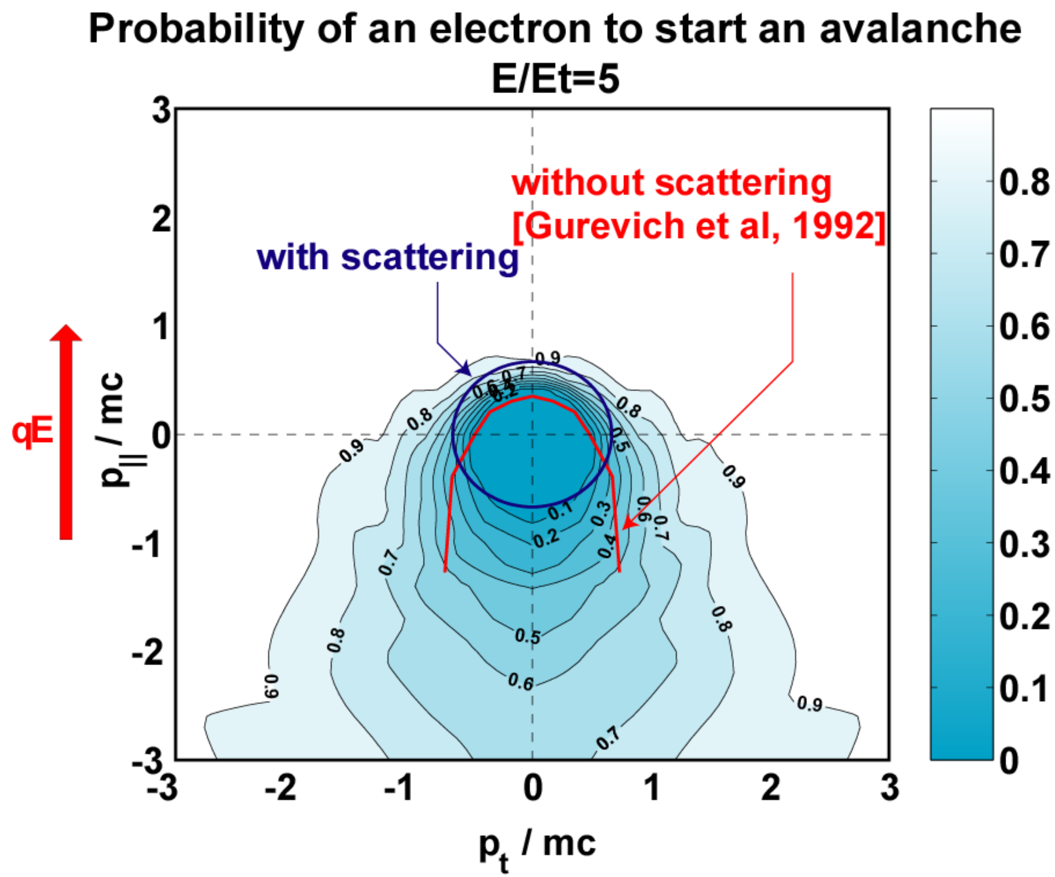
Altitude above which B becomes important

- For f_{H0}=1 MHz
- Elastic: $\nu_m=\omega_H$ (ν_m is the momentum transfer rate)
- Inelastic: $F_D/p=\omega_H$ (F_D is the dynamic friction)

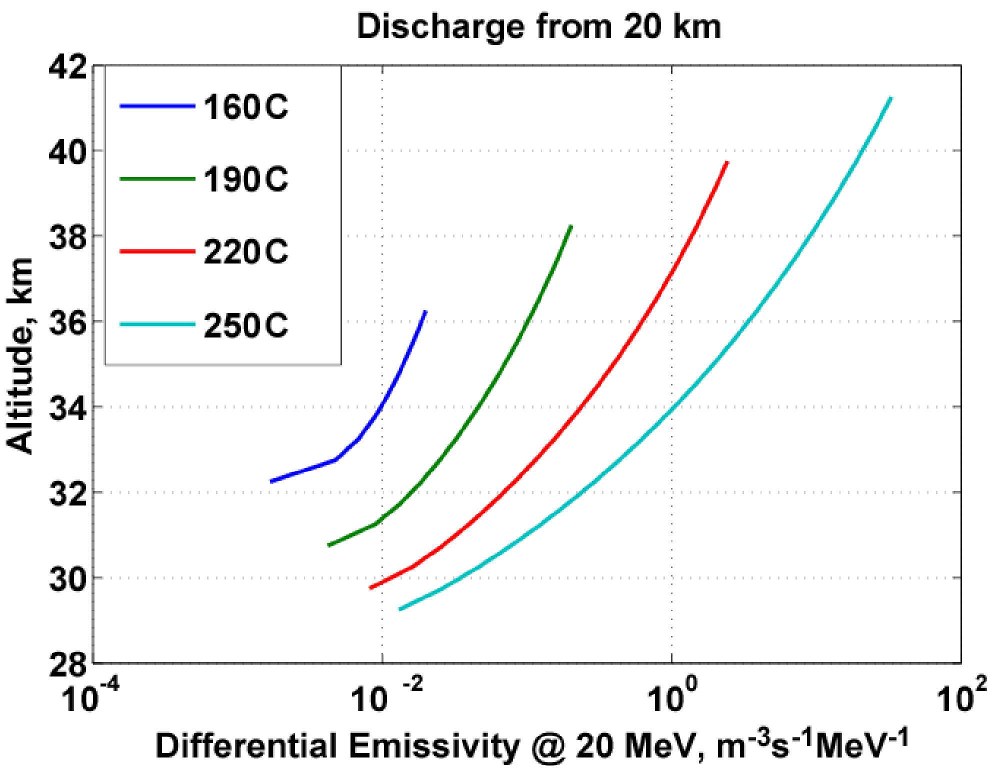


Energy of avalanche-producing electrons

- Low energy electrons are stopped by friction
- Avalanche continues only if $E>E_{min}$
- $E_{min} \sim mc^2/R_0$
- For a uniform avalanche, there are more low-energy electrons => E_{min} is the important energy scale



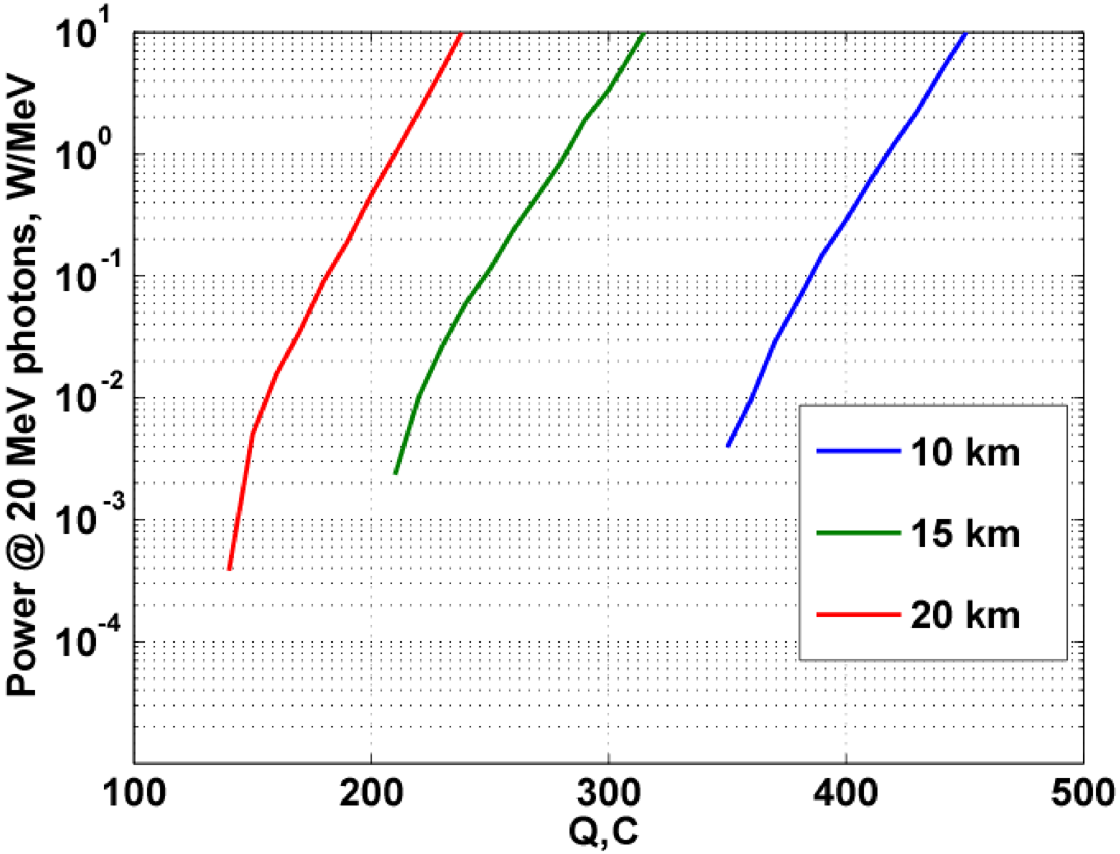
Gamma Ray Emissivity at the Equator



- ε = N N_m v $d\sigma/dE_{ph}$
- $d\sigma/dE_{ph}$ – Heitler's bremsstrahlung cross-section
- Avalanche is suppressed at high altitudes (when B>2E/c)

The power produced in gamma rays at the equator

Integrate over volume, assume ~10km transverse size of the electron beam



Conclusions

- The number of relativistic electrons produced in the avalanche depends very nonlinearly on the electric field
- At the equator, the avalanche has to take place below 40 km