CONVERSION OF TRAPPED UPPER HYBRID OSCILLATIONS AND Z MODE AT A PLASMA DENSITY IRREGULARITY

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ABSTRACT

Analytic expressions are presented for the conversion of electromagnetically driven trapped upper hybrid oscillations and electromagnetic Z mode at a density irregularity in a magnetized plasma, which are valid for arbitrary low resonances of the trapped field. The theory predicts non-Lorentzian skewed shapes of the resonances for the Z mode radiation.

INTRODUCTION

One of the most important effects of electromagnetic pumping of the ionosphere from the ground is the structuring into filamentary plasma irregularities stretched along the geomagnetic field. These striations are central to a number of phenomena, including anomalous absorption [1], stimulated electromagnetic emissions [2], Langmuir turbulence evolution [3], and field-aligned scattering of radio waves [4]. Striations are mainly density depletions of a few percent formed by upper hybrid (UH) oscillations that are trapped in the depletion where they are generated by linear conversion of the pump field on the density gradients [5, 6]. However, the short wave UH oscillations are partially transmitted through the trapping depletion into the long wave electromagnetic Z mode [7, 8]. This Z mode leakage constitutes a large part of the damping that determines the UH amplitude [9]. Also, Z mode leakage appears to have been observed by a sounding rocket in the auroral ionosphere [10, 11] where UH oscillations were excited by electron beams.

Here new analytic expressions are presented for the Z mode amplitude in terms of the eigenfunctions for the trapped UH oscillations. The governing equations are solved in slab geometry by using a scale length separation technique similar to the source approximation first introduced to obtain analytic expressions for mode conversion in unmagnetized plasma [12]. In view of the significance of UH trapping and the strong excitation of the lowest resonances of the trapped field, as shown in numerical studies [13], it is important that our results are derived for arbitrary low resonances, in contrast to previous WKB approximations [7, 8]. This is relevant for the description of the stationary state of striations [5, 6], which include complex nonlinear phenomena of the trapped UH oscillations [14, 13, 15]. Further, our results pave the ground for modeling the interaction of striations, which is fundamental to understand the global plasma structuring in response to electromagnetic pumping as well as the nonlinear mode conversion of the O mode pump wave. Measurements [16] and theories [5, 6] show that striations are isolated filaments, thus constituting a strongly inhomogeneous distribution of plasma turbulence. However, the considerable Z mode leakage suggests that many striations must interact and be excited together, so that the effective leakage is reduced due to the Z mode influx from surrounding striations [17].

BASIC EQUATIONS

We consider the time harmonic linear wave equation for high frequency waves, i.e., only for electron dynamics,

$$\nabla^2 \mathbf{E} - \nabla(\nabla\cdot\mathbf{E}) + k_0^2\,\mathbf{D}/\epsilon_0 = 0\,, \tag{1}$$

where $k_0 \equiv \omega/c$ (ϵ_0 is the permittivity of vacuum and c is the vacuum light speed). The time dependence of the electric field $\mathbf{E}$ and displacement $\mathbf{D}$ is $\propto \exp(-i\,\omega\,t)$ where ω is the pump frequency. The relation between $\mathbf{D}$ and $\mathbf{E}$ has to take into account corrections for $\beta \equiv v_{\mathrm{T}}/c > 0$ (v_{T} is the electron thermal speed) in order to describe UH waves, as well as for small deviations in the background plasma density $n_0(\mathbf{r}) = n_0[1+\eta(\mathbf{r})]$ where $|\eta| \ll 1$. A useful model of the dielectric response can be derived from the fluid equations

$$\mathbf{D}/\epsilon_0 = \kappa'\,\mathbf{E} + \gamma\,(\beta/k_0)^2\,\sigma/\sigma_\parallel\,\nabla(\nabla\cdot\mathbf{E})\,, \tag{2}$$

where $\sigma_\parallel \equiv i\,\epsilon_0\,\omega\,X$, $X \equiv (\omega_{\mathrm{p}}/\omega)^2$ (ω_{p} is the electron plasma frequency), $\gamma = 3$ is the appropriate adiabatic index, $\kappa' = \kappa + \delta\kappa$, $\delta\kappa = -X\,\eta\,\sigma/\sigma_\parallel$, and σ and κ are the cold conductivity and relative permittivity tensor, respectively. The first term in the right-hand side of (2) describes the dielectric response of a cold plasma with a small plasma density variation and the second term is a correction due to $\beta > 0$.

We consider only variations of η in one direction perpendicular to the static ambient magnetic field $\mathbf{B}_0 = B_0\,\hat{\boldsymbol{z}}$ so that $\eta = \eta(x)$. The total electric field is $\mathbf{E} = \mathbf{E}_0 + \mathbf{E}_1$ where $\mathbf{E}_0$ is the O mode pump field and $\mathbf{E}_1$ is the excited field. $\mathbf{E}_1$

varies in the same direction as η, i.e., $\mathbf{E}_1 = \mathbf{E}_1(x)$. The amplitude of the pump wave, which is assumed to propagate parallel to $\mathbf{B}_0$, will decrease along $\mathbf{B}_0$ due to energy transfer to the Z mode and also vary perpendicular to $\mathbf{B}_0$ due to the density irregularity. However, we consider the pump wave to be a constant oscillating field $\mathbf{E}_0 = E_{\mathrm{O}}\,\hat{\boldsymbol{e}}_{\mathrm{O}}$ where the polarization vector is $\hat{\boldsymbol{e}}_{\mathrm{O}} = (\hat{\boldsymbol{x}} - i\,\hat{\boldsymbol{y}})/\sqrt{2}$. By writing (1) and (2) in rectangular coordinates one finds that the equation for E_z is not coupled to E_x or E_y. E_z describes O mode waves propagating perpendicular to $\mathbf{B}_0$ and will not be considered further. The E_x and E_y components are coupled and describe the X and Z mode. The equations are

$$(k_0^{-2}\,\Lambda\, d^2/dx^2 + \kappa')\mathbf{E}_1 = -\delta\kappa\,\mathbf{E}_0\,, \quad \text{where} \quad \Lambda \equiv \tilde{\beta}^2\begin{pmatrix} 1 & 0 \\ i\,Y & \tilde{\beta}^{-2} \end{pmatrix}, \tag{3}$$

$\tilde{\beta}^2 \equiv \gamma\,\beta^2/(1 - Y^2)$, and $Y \equiv \omega_{\mathrm{c}}/\omega$ (ω_{c} is the electron gyro frequency). In order to decouple (3) outside the irregularity we express $\mathbf{E}_1$ in field coordinates corresponding to the wave modes in the homogeneous plasma outside the irregularity. We introduce the normalized polarization vectors $\hat{\boldsymbol{e}}_{\mathrm{X}}$ and $\hat{\boldsymbol{e}}_{\mathrm{Z}}$ corresponding to the X and Z mode polarization, respectively, as basis vectors. The polarization vectors are, except for a phase factor, determined by $\kappa\,\hat{\boldsymbol{e}}_I = N_I^2\,\Lambda\,\hat{\boldsymbol{e}}_I$ for $I \in \{\mathrm{X}, \mathrm{Z}\}$ where the refractive indices N_{X} and N_{Z} are roots of the biquadratic equation $|\kappa - N^2\,\Lambda| = 0$. For $X > 1 - Y^2$ $N_{\mathrm{X}}^2 \approx N_{\mathrm{UH}}^2$ where $N_{\mathrm{UH}}^2 \equiv (1 - Y^2 - X)/(\gamma\,\beta^2)$ is the refractive index for an UH wave in the electrostatic approximation and $N_{\mathrm{Z}}^2 \approx 1 - X(1 - X)/(1 - Y^2 - X)$ is the refractive index for a Z mode wave in the cold plasma approximation. We take $\mathbf{E}_1 = E_{\mathrm{X}}\,\hat{\boldsymbol{e}}_{\mathrm{X}} + E_{\mathrm{Z}}\,\hat{\boldsymbol{e}}_{\mathrm{Z}}$ where E_{X} and E_{Z} is the amplitude of the X and Z mode, respectively. In this representation of $\mathbf{E}_1$ equations (3) are transformed to

$$\mathcal{L}_{\mathrm{X}}\,E_{\mathrm{X}} - Q_{\mathrm{XZ}}\eta\,E_{\mathrm{Z}} = Q_{\mathrm{XO}}\eta\,E_{\mathrm{O}} \tag{4}$$
$$\mathcal{L}_{\mathrm{Z}}\,E_{\mathrm{Z}} + Q_{\mathrm{ZX}}\eta\,E_{\mathrm{X}} = -Q_{\mathrm{ZO}}\eta\,E_{\mathrm{O}} \tag{5}$$

which explicitly shows the two modes to be coupled only due to $\eta \neq 0$. The operators are $\mathcal{L}_{\mathrm{X}} \equiv -k_{\mathrm{X}}^{-2}\,d^2/dx^2 - (1 + Q_{\mathrm{XX}}\eta)$ and $\mathcal{L}_{\mathrm{Z}} \equiv k_{\mathrm{Z}}^{-2}\,d^2/dx^2 + 1 + Q_{\mathrm{ZZ}}\eta$ where $k_I \equiv k_0\,N_I$. The coupling constants are $Q_{IJ} = \delta_{IJ} - N_I^{-2}\,\hat{\boldsymbol{e}}_I^\dagger\,P\,\Lambda^{-1}\hat{\boldsymbol{e}}_J$ The matrix $P \equiv (2\cdot 1_2 - \hat{\boldsymbol{e}}_{\mathrm{X}}\hat{\boldsymbol{e}}_{\mathrm{X}}^\dagger - \hat{\boldsymbol{e}}_{\mathrm{Z}}\hat{\boldsymbol{e}}_{\mathrm{Z}}^\dagger)/(1 - |\hat{\boldsymbol{e}}_{\mathrm{X}}^\dagger\hat{\boldsymbol{e}}_{\mathrm{Z}}|^2)$ has the properties $\hat{\boldsymbol{e}}_I^\dagger\,P\,\hat{\boldsymbol{e}}_J = \delta_{IJ}$ and is introduced because $\hat{\boldsymbol{e}}_{\mathrm{X}}$ and $\hat{\boldsymbol{e}}_{\mathrm{Z}}$ are not mutually orthogonal (1_2 is the 2×2 unity matrix). The boundary conditions correspond to outward propagating Z mode and vanishing X mode at infinity, i.e., $E_{\mathrm{X}} \to 0$ and $E_{\mathrm{Z}} \propto \exp(i\,k_{\mathrm{Z}}\,|x|)$ as $|x| \to \infty$.

ANALYSIS

The wave equations (4) and (5) and the boundary conditions can be written as two coupled integral equations. Multiplying (4) and (5) by an appropriate Green's function and integrating gives

$$E_{\binom{\mathrm{X}}{\mathrm{Z}}}(x) = (\pm)\,Q_{\binom{\mathrm{XZ}}{\mathrm{ZX}}}\int_{-\infty}^{\infty} G_{\binom{\mathrm{X}}{\mathrm{Z}}}(x, x_1)\eta(x_1)\,E_{\binom{\mathrm{Z}}{\mathrm{X}}}(x_1)\,dx_1 + E^{(0)}_{\binom{\mathrm{X}}{\mathrm{Z}}} \tag{6}$$

The system of coupled integral equations (6) can be written as two uncoupled Fredholm integral equations of the second kind, which are

$$E_{\binom{\mathrm{X}}{\mathrm{Z}}}(x) - \int_{-\infty}^{\infty} K_{\binom{\mathrm{X}}{\mathrm{Z}}}(x, x_1)\,E_{\binom{\mathrm{X}}{\mathrm{Z}}}(x_1)dx_1 = E^{(0)}_{\binom{\mathrm{X}}{\mathrm{Z}}} + E^{(1)}_{\binom{\mathrm{X}}{\mathrm{Z}}} \tag{7}$$

where the kernels K_{X} and K_{Z} are

$$K_{\binom{\mathrm{X}}{\mathrm{Z}}}(x, x_1) = -Q_{\mathrm{XZ}}\,Q_{\mathrm{ZX}}\int_{-\infty}^{\infty} G_{\binom{\mathrm{X}}{\mathrm{Z}}}(x, x_2)\,G_{\binom{\mathrm{Z}}{\mathrm{X}}}(x_2, x_1)\,\eta(x_1)\,\eta(x_2)\,dx_2\,. \tag{8}$$

The Green's functions G_{X} and G_{Z} satisfy the uncoupled equations $\mathcal{L}_{\mathrm{X}}G_{\mathrm{X}}(x, x_1) = \delta(x - x_1)$ and $\mathcal{L}_{\mathrm{Z}}G_{\mathrm{Z}}(x, x_1) = \delta(x - x_1)$. The boundary conditions are $G_{\mathrm{X}}(x, x_1) \to 0$ and $G_{\mathrm{Z}}(x, x_1) \propto \exp(i\,k_{\mathrm{Z}}\,|x|)$ as $|x| \to \infty$. The source terms $E_{\mathrm{X}}^{(0)}$, $E_{\mathrm{Z}}^{(0)}$, $E_{\mathrm{X}}^{(1)}$, and $E_{\mathrm{Z}}^{(1)}$ in (7) are given by

$$E^{(0)}_{\binom{\mathrm{X}}{\mathrm{Z}}}(x) = (\pm)Q_{\binom{\mathrm{XO}}{\mathrm{ZO}}}\int_{-\infty}^{\infty} G_{\binom{\mathrm{X}}{\mathrm{Z}}}(x, x_1)\,\eta(x_1)\,E_{\mathrm{O}}\,dx_1 \tag{9}$$
$$E^{(1)}_{\binom{\mathrm{X}}{\mathrm{Z}}}(x) = (\pm)Q_{\binom{\mathrm{XZ}}{\mathrm{ZX}}}\int_{-\infty}^{\infty} G_{\binom{\mathrm{X}}{\mathrm{Z}}}(x, x_1)\,\eta(x_1)\,E^{(0)}_{\binom{\mathrm{Z}}{\mathrm{X}}}(x_1)\,dx_1\,. \tag{10}$$

One can interpret $E_I^{(0)}$ as the zeroth order Born approximation of (6) which describes direct excitation of X and Z mode waves by scattering the pump wave off the density irregularity. Similar to the interpretation of $E_{\mathrm{Z}}^{(0)}$ as direct excitation,

$E_{\rm Z}^{(1)}$ can be interpreted as excitation of Z mode waves in a two step process. First X mode waves are excited as described by $E_{\rm X}^{(0)}$ which is followed by that $E_{\rm X}^{(0)}$ is scattered off the irregularity to excite Z mode waves. Contrary to the equations (3) together with the boundary conditions, the equations (7) are uncoupled and well suited for approximations.

In what follows, the treatment is restricted to the case with one isolated density irregularity. The isolated density depletion has a characteristic width $L_\perp$ perpendicular to $\mathbf{B}_0$ and is centered around $x = \xi$. No further assumptions about η are necessary and all formal calculations can be made without assuming any specific shape of the irregularity. As $G_{\rm Z}$ changes on length scales $L_{\rm Z} \equiv k_{\rm Z}^{-1}$ and η is essentially only different from zero for $|x-\xi| < L_\perp$ one can approximate the integral $\int_{-\infty}^{\infty} f(x_2)\, G_{\rm Z}(x_1,x_2)\, \eta(x_2)\, dx_2 \approx G_{\rm Z}(x_1,\xi) \int_{-\infty}^{\infty} f(x_2)\, \eta(x_2)\, dx_2$ for any function $f(x)$, assuming $L_\perp \ll L_{\rm Z}$. With this approximation and the definition (9) of $E_{\rm X}^{(0)}$ the kernel $K_{\rm X}$ in (8) is

$$K_{\rm X}(x,x_1) \approx -\frac{Q_{\rm XZ}\, Q_{\rm ZX}}{Q_{\rm XO}}\, G_{\rm Z}(x_1,\xi)\, \eta(x_1)\, E_{\rm X}^{(0)}(x)/E_{\rm O}\,. \tag{11}$$

Notice that $K_{\rm X}$ can approximatively be written as a product of one function of x and one of x_1, which thus constitutes a degenerate kernel approximation. With the degenerate kernel (11) it is trivial to solve (7). Particularly, far away from the irregularity ($|x-\xi| \gg L_\perp$)

$$E_{\rm Z}/E_{\rm O} \approx -\left(Q_{\rm ZO}\, \hat\eta + Q_{\rm ZX} \int_{-\infty}^{\infty} \eta\, E_{\rm X}^{(0)}/E_{\rm O}\, dx_1\right) \left(1 + \frac{Q_{\rm XZ}\, Q_{\rm ZX}}{Q_{\rm XO}}\, G_{\rm Z}(\xi,\xi) \int_{-\infty}^{\infty} \eta\, E_{\rm X}^{(0)}/E_{\rm O}\, dx_1\right)^{-1} G_{\rm Z}(x,\xi)\,. \tag{12}$$

Similarly, by using the length scale separation $G_{\rm Z}(x,\xi)$ can be approximated

$$G_{\rm Z}(x,\xi) \approx \frac{i\, k_{\rm Z}/2}{1 + i\, Q_{\rm ZZ}\, k_Z\, \hat\eta/2}\, \exp(i\, k_{\rm Z}\, |x-\xi|) \tag{13}$$

where $\hat\eta \equiv \int_{-\infty}^{\infty} \eta\, dx_1$. The solution (12) do not involve $G_{\rm X}$ explicitly. $E_{\rm Z}$ can instead be constructed directly from $E_{\rm X}^{(0)}$. By using the equation for $G_{\rm X}$ and (9) one finds

$$(\mathfrak{L}_{\rm UH} + \Delta) E_{\rm X}^{(0)} = Q_{\rm XO}/Q_{\rm XX}\, \eta\, E_{\rm O} \tag{14}$$

where $\mathfrak{L}_{\rm UH} \equiv \lambda^2 d^2/dx^2 - \eta$, $\lambda^2 \equiv -k_{\rm X}^{-2}\, Q_{\rm XX}^{-1} \approx \gamma\, \lambda_{\rm D}^2\, X$ ($\lambda_{\rm D}$ is the Debye length), and $\Delta \equiv -Q_{\rm XX}^{-1} \approx 1 - Y^2 - X$. The solution to (14) can be written as a superposition of eigenfunctions to $\mathfrak{L}_{\rm UH}$. If the contribution from the continuous spectrum is neglected the solution is [13]

$$E_{\rm X}^{(0)}/E_{\rm O} \approx \frac{Q_{\rm XO}}{Q_{\rm XX}} \sum_{n=0}^{M} \frac{\eta_n\, \psi_n(x)}{\Delta - \Delta_n} \tag{15}$$

where $\eta_n = \int_{-\infty}^{\infty} \eta\, \psi_n^*\, dx_1$. The normalized eigenfunctions ψ_n and eigenvalues Δ_n ($\min \eta < \Delta_0 < \Delta_1 < \cdots < \Delta_M < 0$) are determined by the equation $(\mathfrak{L}_{\rm UH} + \Delta_n)\psi_n = 0$ and the boundary conditions $\psi_n \to 0$ as $|x| \to \infty$. $E_{\rm Z}$ can be expressed in terms of η_n and Δ_n by combining (12) and (15), for $|x-\xi| \gg L_\perp$

$$E_{\rm Z}/E_{\rm O} \approx -\left(\frac{i\, Q_{\rm ZO}\, k_{\rm Z}\, \hat\eta/2}{1 + i\, Q_{\rm ZZ}\, k_{\rm Z}\, \hat\eta/2} - i\, \frac{Q_{\rm XO}}{Q_{\rm XZ}} \sum_{n=0}^{M} \frac{\nu_n/2}{\Delta - \Delta_n}\right) \left(1 - i \sum_{n=0}^{M} \frac{\nu_n/2}{\Delta - \Delta_n}\right)^{-1} \exp(i\, k_{\rm Z} |x-\xi|) \tag{16}$$

where $\nu_n \equiv -(k_{\rm Z}\, |\eta_n|^2\, Q_{\rm XZ}\, Q_{\rm ZX}/Q_{\rm XX})(1 + i\, Q_{\rm ZZ}\, k_{\rm Z}\, \hat\eta/2)^{-1}$. The real part of ν_n can be interpreted as the width of the resonance at Δ_n and the imaginary part as a shift of the resonance frequency.

RESULTS AND CONCLUSIONS

In order to demonstrate the use of the solution (16) we consider $\eta(x) = -\tilde\eta\, \mathrm{sech}^2(x/L_\perp)$ where $\tilde\eta > 0$ characterizes the depletion amplitude and ψ_n are given in terms of the hypergeometric function. Figure 1 shows $|E_{\rm Z}/E_{\rm O}|^2$ as a function of Δ. As (16) indicates, $|E_{\rm Z}/E_{\rm O}|$ is particularly large near the resonances Δ_n and the resonances are not purely Lorentzian. Instead they are increasingly skewer for increasing resonance number, which is due to interference of Z mode leakage from non-resonant UH modes. The width and skewness of the resonances can be used for experimental verification of Z mode leakage from trapped UH oscillations. Figure 1 also shows the WKB approximation [7, 8] of the same quantity. The WKB solution, which was derived in an approximation that excludes the lowest resonances, reproduces nevertheless the

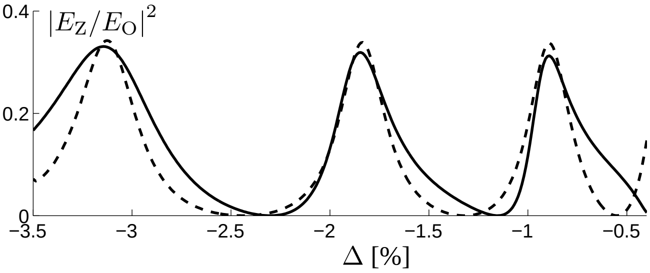

Figure 1: $|E_{\rm Z}/E_{\rm O}|^2$ as a function of Δ from (16) (solid line) and from the WKB approximation (dashed line). The WKB solution was calculated using a slightly improved density correction compared to Ref. [7, 8]. The parameters are $\beta = 1.5 \cdot 10^{-3}$, $L_\perp = 1.0$ m, $\tilde{\eta} = 3.5\%$, $\omega_{\rm p}/2\pi = 6$ MHz, and $\omega_{\rm p}/\omega_{\rm c}$ =4.5.

position of the resonances, but not the width of the lowest resonance and the skewness. The skewness is not described by the WKB solution since the interaction between the UH and Z mode is assumed to be confined to the electrostatic cutoffs where the inner UH and the outer Z mode solutions are matched.

The authors gratefully acknowledge valuable discussion with George Morales and Yakov Istomin. J.O.H. was supported by the Advanced Instrumentation and Measurements graduate school. T.B.L. was supported by the Swedish Research Council.

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